# Comparison of RC-model and FEM-model for a PCM-plate storage including free convection

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# Highlights

- Latent heat storage based on FracTherm®-plates using D-mannitol as PCM
- Development of simplified RC-model including free convection during PCM melting
- Comparison of simplified RC-model to numerical FEM-model
- Validation of FEM-model with experimental data
- Deviation of outlet fluid temperature of RC-model and FEM-model of 0.62 K

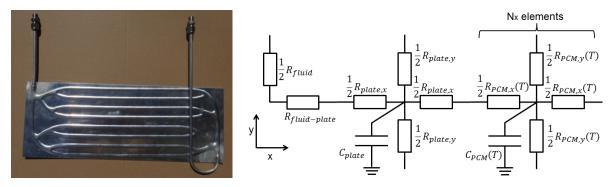
# Abstract

Latent heat storage is one option to increase the efficiency and reduce CO<sub>2</sub>-emissions of process heat applications in the temperature range between 100 °C and 250 °C. In the present study a latent heat storage system based on flown through heat exchanger plates according to the Frac-Therm®-design is examined. The sugar alcohol D-mannitol is used as PCM. A simplified capacity resistor (RC) simulation model is developed. In contrast to common RC-models, this model can simulate free convection of the storage material in liquid phase during charging. Additionally, a detailed physical simulation model based on the Finite-Element-Method is developed and validated with measured data. A verification of the new RC-model and the FEM-model is carried out. The mean deviation of the outlet fluid temperature between both models is 0.62 K. The mean deviation of the PCM-temperature is 0.85 K. Due to the 20 to 30 times shorter simulation time, the RC-model is well suited for dimensioning and optimizing plate type heat exchangers for latent heat storages.

# Keywords

Phase change material, D-mannitol, FracTherm<sup>®</sup>, RC-model, FEM-model, free convection

# Graphical abstract



# Nomenclature

## Symbols

Ż	W	Heat flow	
$c_p$	J/(kg K)	Specific heat capacity	
'n	kg/s	Mass flow	
<i></i>	W/m <sup>2</sup>	Heat flux	
ġ	W/m <sup>3</sup>	Volumetric heat flux	
h	J/kg	Specific Enthalpy	
Α	m <sup>2</sup>	Area	
Α	1	Aspect ratio	
С	J/K	Heat capacity	
D	К	Mean deviation	
F	N, N/m <sup>3</sup>	Force	
Н	m	Height	
R	K/W, Ω	Resistance	
Ra	1	Rayleigh-Number	
Т	К	Temperature	
V	m <sup>3</sup>	Volume	
b	m	width	
d	m	Thickness	
f	1	Fraction	
g	m/s <sup>2</sup>	Gravitational constant	
l	m	Length	
т	kg	Mass	
n	1	Number	
p	Ра	Pressure	
t	S	Time	

v	m/s	Velocity
α	W/(m <sup>2</sup> K)	Heat transfer coefficient
β	1/m	Volumetric expansion coefficient
η	Pa s	Dynamic Viscosity
λ	W/(m K)	Heat Conductivity
ρ	kg/m <sup>3</sup>	Density
$\epsilon$	1	convective enhancement factor

## Abbreviations

one-dimensional
two-dimensional
three-dimensional
Computational Fluid Dynamics
Differential Scanning Calorimeter
Finite-Element-Method
Potassium nitrate
Sodium nitrate
Phase Change Material
Polytetrafluoroethylene
Resistance Capacity

# Subscripts

Cond	Conduction
Conv	Convection
crit	critical
FEM	Finite-Element-Method
in	Inlet
1	Liquid
m	Melt
meas	Measurement
out	outlet
РСМ	Phase Change Material
S	Solid
sim	Simulation
tot	Total

# 1 **1 Introduction**

- 2 One of the major challenges nowadays is the reduction of worldwide greenhouse gas emissions,
- 3 thereby faciliatating the protection of the climate. One measure to reach this goal is to reduce and
- 4 make full use of waste heat. In industrial processes, the utilization of thermal storage can, for ex-
- 5 ample, contribute to the peak-load shifting [1, S. 4], and consequently reduce the generation of
- 6 waste heat.
- 7 In process heat applications, defined in the temperature range between 100 °C and 250 °C [2, S.
- 8 536], water can only be used as pressurised heat storage medium. These storages are expensive
- 9 because the vessels are required to tolerate high pressure [3, S. 48]. Alternative pressureless heat
- 10 storages use, for example, thermal oil, rocks, concrete or ceramics as storage material. However,
- 11 these materials have a low specific heat capacity, and thus big storage volumes are needed, given
- 12 a small temperature application range.
- 13 For applications which require high heat capacity and small temperature application ranges, la-
- 14 tent heat storage is an attractive alternative. The latent heat storage tends to be achieved using
- 15 the so-called Phase Change Materials (PCM) as storage media, due to their high thermal energy
- 16 storage density within an almost constant temperature.
- 17 Process heat storages are usually short term storages (i.e. storages that lasts for between hours
- 18 and days) and tend to deliver higher power [2, S. 536]. In addition, process heat applications fea-
- 19 ture the requirements of different temperatures and thermal power for individual processes,
- 20 when applying heat storage. Therefore, a simple storage concept needs to be adapted to satisfy
- 21 the various storage requirements under different processes.
- 22 Research on many different heat exchanger concepts for latent heat storages in the temperature 23 range between 100 °C and 250 °C have already been extensively investigated: Johnson et al. [4],[5] 24 developed a plate heat exchanger using an eutectic mixture of KNO<sub>3</sub> and NaNO<sub>3</sub> as PCM. The au-25 thors examined different heat transfer structures to adapt the storage power to different applica-26 tions. Saeed et al. [6] used parallel aluminium plates as heat exchanger and carried out experi-27 ments with different plate distances. Agyenim et al. [7] examined fin-and-tubes heat exchangers 28 with the sugar alcohol erythritol as PCM. Gil et al. [8],[9] present a fin-and-tubes latent heat stor-29 age using hydrochinon as PCM. Peiró et al. [10] investigated the use of hydrochinon and D-manni-30 tol as PCM in a fin-and-tubes heat exchanger. Steinmann et al. [11] studied a fin-and-tubes heat 31 exchanger with graphite or aluminium fins and NaNO<sub>3</sub> or eutectic mixtures as PCM. Bayón et al. 32 [12] further tested one of these heat exchangers in a test platform for solar thermal power plants. 33 Zauner et al. [13] described a fin-and-tubes heat exchanger using HDPE as PCM. Shon et al. [14] 34 presented a fin-and-tubes heat exchanger with the sugar alcohol xylitol as PCM. Wang et al. [15] 35 investigated two shell-and-tubes heat exchangers using A 153 as PCM (melting temperature be-36 tween 151 and 155 °C). They examined the heat transfer-enhancement techniques by adding ho-37 mogenous and gradient metal foams. Anish et al. [16] experimentally examined a horizontal shell-38 and-multi-finned tube latent heat storage using erythritol as PCM and analyzed the effect of free 39 convection during melting. Further storage concepts in the temperature range suitable for process 40 heat applications use microcapsules and composite materials [17] or 3D-wire structures in com-41 bination with salts and their eutectic mixtures [18]. Zipf et al. [19] reported an active latent heat 42 storage concept based on a screw heat exchanger using an eutectic mixture of NaNO<sub>3</sub> and KNO<sub>3</sub> as 43 PCM. Pointner at al. [20],[21] also described an active storage concept by transporting the storage

- 44 material across the heat transfer surface. Kaizawa et al. [22] studied a direct-contact latent heat
- 45 storage using the sugar alcohol erythritol as PCM. Mosleh et al. [23] tested the influence of a latent
- 46 heat storage using H250, NaNo<sub>3</sub>, KNO<sub>3</sub> and KOH as PCM in a solar power plant.

47 Based on the above literature review, many different heat exchanger types have been investigated,

48 however, using of Fractherm<sup>©</sup> heat exchanger plates for latent heat storage has not been reported.

49 As one of the novelties, this study developed and tested the heat storage using this type of heat

- 50 exchanger plates. These plates have a characteristic fluid-channel design, which enables a uniform
- volume flow distribution and a low pressure-drop due to the flow optimization in branched fluid
- 52 distribution, in comparison to standard manifold design with parallel fluid distribution [24]. The
- bionic FracTherm<sup>®</sup>-design was developed at Fraunhofer ISE [24] and it is the first time that it was
   utilized and applied to developed heat exchanger plates in PCM-storages. Due to the fabrication
- utilized and applied to developed heat exchanger plates in PCM-storages. Due to the fabrication
  by roll bonding, the channels and the aluminium sheet form a union and thus lead to a good heat
- 56 transfer [24]. For different storage requirements in process heat applications, different parame-
- 57 ters such as the plate distance, the plate size and the number of channels, etc. can be adjusted,
- 58 thereby providing high flexibility for the design process.

59 Different from the phase change temperatures higher than 200 °C where molten salts can be used,

60 many PCMs with phase change temperatures in the range between 100 °C and 250 °C are still un-

- der research, and there is a strong demand for more materials available for this temperature
   range. Due to their melting ranges and high storage densities, sugar alcohols are of interest for
- 63 this temperature range. Unfortunately, these materials show poor thermal stability. Thus, the
- 64 studies in public domain in this sector mainly focus on the stabilization and stability enhancement
- of these materials [25]–[27]. For instance, some researchers tried to improve the reliablity by
- 66 microencapsulation [28]. Some other materials for this temperature range are also under re-
- 67 search, however, they present disadvantages from various aspects [29]. Accordingly, on one hand
- there are only a few materials available for the desired temperature range while many of them
- 69 show disadvantages such as poor stability which rises a necessity of further research; on the other

hand, a powerful and cost-effective heat exchanger system is also necessary to enable the efficient

- 71 heat extraction or storage for industrial process applications.
- FEM-simulation is a very useful and powerful tool for heat exchanger development and optimiza-
- tion. The influential factors which affect the heat transfer between the heat transfer fluid and the
- storage material on a small scale, e.g. different heat exchanger surface structures, can be well in-
- vestigated using this tool. However, for larger scale simulations, e.g. optimization of heat ex-
- 76 changer systems or thermal energy storage systems, it has the drawback of high computation cost
- or long simulation durations. Therefore, a more suitable tool with high simplicity and less compu-
- 78 tation cost is of high necessity.
- Descriptions and summeries on many different simplified simulation models for PCM storages can
  be easily found in literature [30],[31]. For instance, Pointner et al. [32] employed four different
- 81 models to simulate a PCM plate storage, whose modelling results were further compared and nat-
- ural convection of the PCM was considered in these numerical models. Arena et al. [33] modelled
- a storage using D-mannitol as PCM under three different geometrical configurations. A FEM-model
- 84 was utilized in which both heat conduction and free convection were considered.
- 85 Simplified models for PCM storages were usually developed based on resistances and capacities
- 86 (RC). Mirzaei et al. [34], for instance, described a one-dimensional analytical RC-network model
- 87 for PCM-storages in building applications. The model featured phase-status-dependent capacities

- and resistors. Several other models for various applications can be found in literature [35] [36]
- 89 [37] [38] [39]. All these publications developed RC-models, however, did not consider the de-
- 90 creasing of the thermal resistance due to free convection of the liquid PCM during the melting
- 91 process. For applications where this effect influences the heat transfer significantly, free convec-92 tion is of great importance and should be properly modelled for the sake of appropriate system
- 93 sizing and design. In this study, a simplified RC-model was developed with considering free con-
- 94 vection during the melting process in rectangular PCM gaps, based on correlations proposed by
- 95 Vogel et al. [41].
- 96 Farid et al. [42] simulated a storage made of PCM-plates and using air as heat transfer fluid. In this
- 97 numerical study, free convection was modelled by introducing an *effective heat conductivity* that
- 98 depended on the Rayleigh-number and empirical constants related to the storage dimensions. In
- 99 contrast, the RC-model developed in this study does not require any adaption of empirical con-
- 100 stants depending on the storage dimensions.
- 101 In addition to the RC-model, a detailed physical simulation model based on the finite element
- 102 method (FEM) was developed for the sake of model comparison and examination. A plate type
- 103 heat exchanger with considering free convection of the liquid PCM during melting was modelled
- 104 using the RC-model, which was then further verified using the FEM-model. Compared to real 3D
- 105 computation, the FEM-model was significantly simplified by coupling a 2D simulation of the PCM
- 106 gap between two plates with a 1D simulation for one fluid channel of the heat exchanger plate.
- 107 The model was developed based on an approach proposed and validated for a fin-and- tubes heat
- 108 exchanger modelling [43],[44].
- 109 Finally, the accuracy of the RC-model compared to that of the FEM-model was assessed and dis-
- 110 cussed. Both simulation models, the validation using experimental measured data, and the com-
- 111 parison between them are presented in the present paper.

# 112 **2** Description of the system

To provide data for model validation, an experimental characterisation of the storage was carriedout. The storage and the experimental setup are described in this section.

## 115 **2.1 Description of the FracTherm®-plate storage**

- The left picture of Figure 1 shows a FracTherm<sup>®</sup>-plate used in the storage. It was made of aluminium sheet with a based thickess of 1.2 mm, and had a plane dimension of 400 mm x 150.5 mm. A series of channels were embedded within the aluminium sheet, leading to bulges with an overall thickness of 3 mm. The inlet and outlet connections to these channels were made of aluminium tubes with an outer diameter of 6 mm. Thermal oil Therminol 66<sup>®</sup> [45] was used as the heat trans-
- 121 fer fluid, and can be drawn through the channels via the two connections.



- Figure 1: left: FracTherm®-plate with connection tubes; right: top view of the storage envelope including five
   FracTherm®-plates
- 124 The right part of Figure 1 presents a top view of the storage vessel. It consisted of a container
- 125 made of stainless steel, and five FracTherm®-plates installed in parallel. These plates were in-
- 126 stalled with a distance of 25 mm by placing PTFE spacers between them at the right and the left
- 127 sides. It is worthwhile to mention that the distance between the lateral plates and the container
- 128 wall was a half of the distance between plates (12.5 mm). This storage vessel was able to contain
- the PCM with a total volume of 10.76 l, while providing a total heat transfer area of 0.6 m<sup>2</sup>.
- 130 The connection tubes of the plates were connected in parallel using a distributor and a collector.
- 131 The whole vessel was thermally insulated using rock wool with a thickness of 12 cm [46].

## 132 **2.2 PCM**

- 133 The sugar alcohol D-mannitol with a purity of 99.2 % was used as PCM. The relevant material
- 134 properties are summarised in Table 1. 13.0 kg of D-mannitol were filled into the storage vessel in
- 135 liquid state.

Property	Value	Remark	Source
Melting tempera- ture / °C	164.1	Onset	[47]
Melting enthalpy / kJ/kg	300.2		[47]
Density / kg/m <sup>3</sup>	1380 (solid)/1270 (liquid)		[25]
Specific heat ca- pacity / kJ/kgK	1.79 2.75	120 °C 180 °C	own meas- urement
Heat conductivity / W/mK	0.88 0.43	130 °C 190 °C	own meas- urement
Dynamic viscosity / Pa s	0.046	170 °C	own meas- urement
Volumetric coeffi- cient of expansion / K <sup>-1</sup>	1.2 10-4	liquid	based on [25]

136 Table 1: Material properties of *D*-mannitol

137

138 D-mannitol was selected in this study mainly due to its proper melting temperature within the

139 temperature range of interest, even though there is a risk of degradation when heated above its

- 140 melting point [48]–[50]. Also, its high latent heat in initial state together with its high density (see
- 141Table 1) offers a high storage density. In addition, the material is non-flammable, nontoxic, non-
- 142 corrosive and cost-effective [25]. Before and after the experiments, samples of the PCM were
- 143 taken from the storage and the melting enthalpy was measured in DSC Q200 (from TA instruments
- 144 [51]) with a heating rate of 1 K/min and a sample mass of approximately 10 mg.
- 145 Figure 2 shows a picture of the fresh D-mannitol in the storage (left) and that after the experiments
- 146 (right). A clear colour change can be observed after the experiments, in which the material was
- 147 remained above its melting temperature for 18 h.





Figure 2: D-mannitol in the plate heat exchanger before the measurements (left) and after the measurements(right)

- 150 Figure 3 compares the DSC-results in terms of the specific enthalpy and the specific heat capacity
- 151 of the fresh D-mannitol and the post-test D-mannitol after the experiments. It can be found that
- 152 the melting enthalpy decreased, and the correspoding peak became wider while merging towards
- 153 a lower temperature. The melting enthalpy was 286.3 kJ/kg for the fresh D-mannitol and
- 154 237.2 kJ/kg for the post-test samples taken after experiments, resulting in a relative reduction of
- 155 17.1 %.

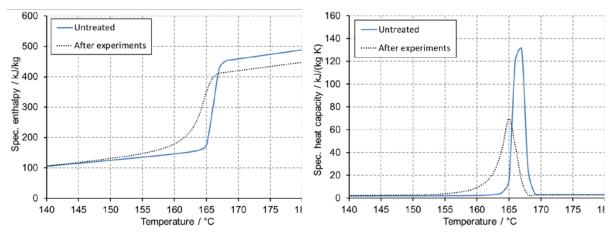


Figure 3: Comparison of the specific enthalpy (left) and the specific heat capacity (right) of untreated D-man nitol as well as the sample taken after the experiments

- 158 As a significant indicator for latent heat storage characteristation, the *Stefan number Ste* was used
- to quantify the ratio of sensible to latent heat capacity, which can be calculated by the following
- 160 equation [41]:

$$Ste = \frac{c_p \Delta T}{h_m} \tag{1}$$

161 For the fresh D-mannitol and an exemplary temperature difference  $\Delta T$  of 20 K, *Ste* reached a value

162 of 0.19. Accordingly, the latent storage capacity was more than 5 times higher than the sensible

163 capacity over this temperature range.

## 164 2.3 Measurement instrumentation

165 The storage was characterised in a test rig at Fraunhofer ISE described in [44]. The mass flow of

- 166 the heat transfer fluid was measured using a Coriolis mass flow meter with an accuracy of 0.11 %
- 167 at nominal flow of 0.8 kg/s [52]. The temperature sensors were calibrated Pt100-sensors with an
- 168 overall accuracy of  $\Delta T = \pm 0.11$  K considering the uncertainty of the reference sensor during cal-
- 169 ibration, the uncertainty of the data acquisition system and the calibration results.
- 170 The left part of Figure 4 illustrates the locations of the inlet and outlet temperature sensors and
- that of the mass flow sensor. The right part of Figure 4 illustrates a cross-sectional view of the
- 172 plates. There were six sensors installed to measure the temperatures of PCM. Three of them were
- placed in the PCM at the height of 20, 75 and 130 mm between plates 2 and 3, respectively. Sym-
- 174 metrically, the other three temperature sensors were placed between plates 3 and 4.

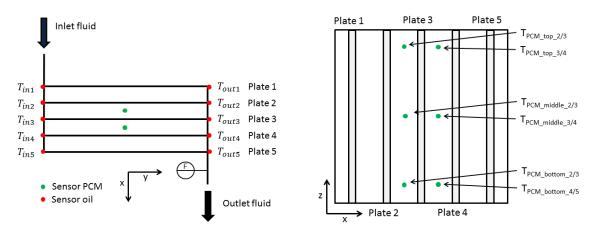


Figure 4: left: top view of the plates with positions of the temperature sensors in the thermal oil, right: cut
 through the plate storage in the middle of the plates in flow direction with positions of the PCM-temperature
 sensors

Further temperature sensors were installed at the exterior surface of the storage vessel and at theexterior surface of the thermal insulation, respectively.

## 180 **2.4 Examination of free convection**

181 The *Rayleigh-number Ra* is a commonly used measure for free convection characterisation. In 182 terms of vertical enclosures, it can be calculated using equation (2) [53].

$$Ra = \frac{g\beta\rho^2 b^3 (T_W - T_{PCM})}{\eta^2} Pr$$
<sup>(2)</sup>

183 The length *b* is the distance between the plate and the solid PCM-layer. Thus, half of the distance 184 between two plates (i.e. 12.5 mm) is considered as this is the theoretical maximum distance be-185 tween a plate and the corresponding solid PCM-layer. The left part of Figure 5 shows the Rayleigh-186 number as a function of the temperature difference for the maxium distance between solid PCM-187 layer and its corresponding heat exchanger plates. According to [53], free convection in liquids 188 starts when *Ra* is higher than 2000. It corresponds to the temperature difference between the 189 heat exchanger surface and the PCM exceeding 4 K. When the temperature difference increases 190 over 10 K, *Ra* is higher than 5000. It is reported that *Ra* of 5000 can be regarded as a threshold, 191 below which free convection during the melting process can be neglected in simulation models 192 [54, S. 307]. Figure 5 right shows the Rayleigh-number over the distance *b* between solid PCM-193 layer and heat exchanger plate at a temperature difference of 10 K. Ra is higher than 2000 starting

- 194 from a distance *b* of 9 mm or more. At a distance of 12.5 mm or more, a value of Ra higher than
- 195 5000 can be reached. Accordingly, it indicates that the free convection can occur and should be
- 196 modelled. It is expected that free convection is only developed weakly because *Ra* is close to the
- 197 boundary values given in literature.

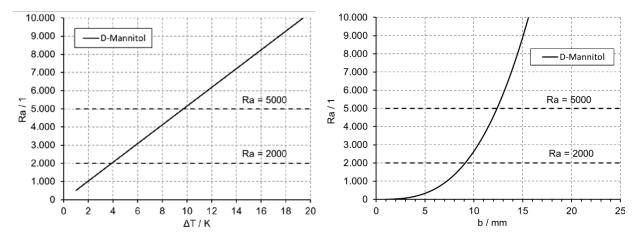


Figure 5: left: Rayleigh-number over ΔT (between plate and solid PCM-layer) at a distance of 12.5 mm, right:
 Rayleigh-number over the distance between plate and solid PCM-layer at a temperature difference of 10 K

## 200 **3 Model description**

In this section the FEM-model and the RC-models are presented and validated respectively verifi-cated.

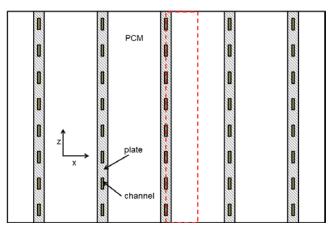
## 203 **3.1 FEM-model**

204 The FEM-model was developed using the software COMSOL Multiphysics® Version 5.3a [55]. Fig-

205 ure 6 illustrates a cross section of the plate storage, in which the small grey rectangulars represent

206 the channels for heat transfer fluid. Due to the fact of symmetry, only the red dashed section high-

lighted in Figure 6 was modelled, while the total hight of the storage was considered for free con-vection simulation.



209

210 Figure 6: Cut through the plate storage in the middle of the plates in flow direction including modelling

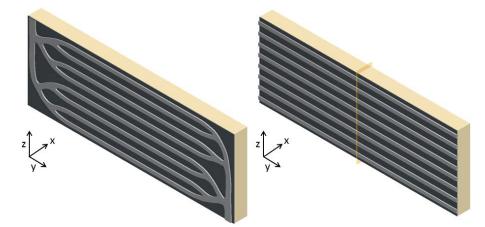
## 211 section marked in dashed red

212 The left picture of Figure 7 shows a 3D-drawing of a FracTherm®-plate with adjacent PCM gap

213 (beige coloured). For the sake of simplification, the channels of the FracTherm®-plates were con-

sidered to be in parallel and the branching of the fluid channels was neglected (right side of Figure

- 215 7). The heat transfer from the plate to the PCM gap was modelled in 2D, indicated by the yellow
- 216 plain in the right picture. The fluid flow in the channels in y-direction was modelled in 1D. The
- aforementioned two models were coupled to each other based on an approach by Wittstadt [43].



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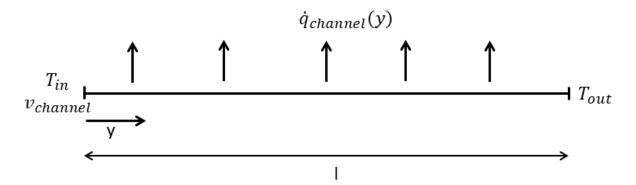
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# Figure 7: Left: 3D-drawing of the FracTherm®-plate with adjacent PCM gap (beige), right: alternative plate with parallel channels incl. cut (yellow) for the 2D-model

- 221 The assumptions and simplification adopted in this study were summarized as follows:
- The heat transfer from the plate to the PCM was modelled in 2D (yellow 2D-section in Figure 7 right). Half a plate and half a PCM gap were modelled in x-direction (Figure 6).
- The fluid channels were assumed to be stretched and in parallel (Figure 7 right). Their
   cross section was assumed to be rectangular [24, S. 13].
- 3. The temperature gradient of the fluid in radial direction of the channels was negligible.
  Thus, the heat transfer fluid was modelled in 1D.
- 228 4. The flow in the channels was fully developed.
- 5. Free convection in the liquid PCM was modelled.
  - a. The flow due to free convection was regarded as laminar.
  - b. The solid phase does not move, thus no sinking of the solid phase during free convection was modelled and thus no contact melting occured.
- A volume change of the PCM during the phase change was neglected. Thus, the density was
  independent from the temperature. The mean value between the density in solid and liquid state was taken as simulation input parameter.
- 7. The other material properties of the PCM which are different in solid and liquid stateswere regarded as constants in solid state and in liquid state, individually.
- 238 8. The material properties of the heat transfer fluid were independent of the temperature.
- 9. The material properties of the heat exchanger were independent of the temperature.

## 240 **3.1.1** One-dimensional model of the fluid in the rectangular channels

Figure 8 shows a sketch of the 1D-model of the fluid flow along the channels in y-direction.





#### 243 Figure 8: 1D-model of the fluid flow in the plate storage in y-direction

Only one of the eight parallel channels was modelled. This simplification is possible because the FracTherm®-channel design leads to a very even distribution of the fluid. This was also confirmed by previous infrared camera measurements which showed that the eight channels had approximately the same temperature profile. The aim of the model was to calculate the volumetric heat flux  $\dot{q}_{channel}$  between the heat transfer fluid and the channel wall, as well as the outlet fluid temperature. Therefore, the temperature of the fluid in the channel can be calculated using equation (3) in COMSOL [56].

$$\rho c_p \frac{\partial T}{\partial t} + \rho c_p v \nabla T + \nabla (-\lambda \nabla T) = 0$$
<sup>(3)</sup>

251 The following boundary conditions were implemented:

- y = 0:  $T_{in}$ ,  $v_{channel}$ : The temperature and the velocity of the heat transfer fluid were implemented either as constant or as time dependent parameters at the channel inlet (Dirichlet boundary condition);
- $y = l: \frac{dT}{dy} = 0$ : The temperature gradient at the channel outlet was zero (Neumann boundary condition); and
- $y: \dot{q}(y) = \dot{q}_{channel}$ : At the whole length of the channel, a convective volumetric heat flux based on the temperature difference between the fluid and the channel wall was implemented corresponding to each node of the channel (Neumann boundary condition)
- 260 The volumetric heat flux  $\dot{q}_{channel}$  between the fluid and the inner channel wall was calculated 261 for each node according to equation (4) with the mean wall temperature  $T_W$  (explanation see 262 chapter 3.1.3) and the local fluid temperature  $T_F(y)$ .

$$\dot{q}_{channel}(y) = \alpha \, \frac{A}{V} \, \left( T_W - T_F(y) \right) \tag{4}$$

263The heat transfer coefficient  $\alpha$  was calculated by a Nusselt-correlation according to [57, S.264151]. The heat transfer area A was determined using equation (5).

$$A = U_{channel} l \tag{5}$$

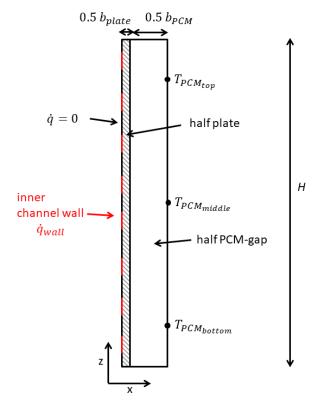
265 The volume *V* was calculated using equation (6).

$$V = b_{channel} d_{channel} l \tag{6}$$

The temperature of the inner channel wall  $T_W$  in equation (4) was calculated in the 2D-model of the plate for every time step and it was used as input value in the 1D-model.

#### 268 **3.1.2 Two-dimensional model with PCM and plate**

- Figure 9 shows a schematic drawing of the 2D-model for PCM and plate, in which the total height
- of the PCM gap and the plate, a half of the PCM gap, and a half of the plate are detailed. The inner
- channel wall is projected onto the plate area. Thus, a red line segment at y = 0 correspond to half
- the channel circumference. Figure 9 also details the locations of three PCM-temperatures meas-
- 273 ured corresponding to that in the experiments ( $T_{PCM_{top}}, T_{PCM_{middle}}, T_{PCM_{bottom}}$ ), which were used
- for data evaluation in the following sections of this study.
- 275 The model developed was used to calculate the temperature distribution in the PCM and the plate.
- 276 The simulation was carried out based on equation (3).



277

#### Figure 9: 2D-model of half a plate and adjacent half PCM gap with boundary conditions

If free convection of the PCM was taken into account, the flow pattern of the melted PCM was regarded as laminar flow. It was calculated according to the *momentum equation* (equation (7))

and the continuity equation (equation (8)) of the Navier-Stokes-equations for incompressible fluids

282 [56].

$$\rho \frac{\partial v}{\partial t} + \rho (v \nabla)v = -\nabla p + \eta \Delta v + F_v$$

$$\rho \nabla v = 0$$
(7)
(8)

The dynamic viscosity  $\eta$  and the volume force  $F_V$  in equation (7) can be determined using the method of the *modified viscosity* according to [58]. In this method the whole PCM is always regarded as liquid even if the temperature is below the melting temperature. In this case a *modified viscosity* forces the liquid to act as a solid by applying an extremely high viscosity. This viscosity is defined by a step function with the following properties:

$$\eta (T) = \begin{cases} 10^6, T < T_m + \Delta T/2 \\ \eta, T \ge T_m + \Delta T/2 \end{cases}$$
(9)

The buoyancy force leading to free convection can be defined by the volume force  $F_V$ . The *bous*sinesq-approximation is applied. Thus, the volume force was calculated according to equation (10):

$$F_V = g \rho_l \beta \left( T - T_m \right) \tag{10}$$

290 The thermal volumetric expansion coefficient  $\beta$  can be calculated using a step function as de-291 scribed in equation (11). Accordingly, the volume force is set to 0 when the PCM is in solid state.

$$\beta = \begin{cases} 0, T < T_m + \Delta T/2 \\ \beta_l, T \ge T_m + \Delta T/2 \end{cases}$$
(11)

The thermal volumetric expansion coefficient  $\beta_l$  in liquid state of the PCM was calculated by equation (12) [59, S. 448]. Thus, a change of the density depending on the temperature in liquid state of the PCM can be considered.

$$\beta_l = -\frac{1}{\rho} \left(\frac{\partial \rho}{\partial T}\right)_p \tag{12}$$

The heat capacity of the PCM was implemented using the *capacity method* [60, S. 282]. Linearlyinterpolated data based on DSC-measurements were used.

The temperature dependent thermal conductivity was calculated according to equation (13) [58], in which B(T) is a *smoothed heaviside function* available in COMSOL.

$$\lambda(T) = \lambda_s + (\lambda_l - \lambda_s) B(T)$$
(13)

299 The material properties of the aluminium heat exchanger plates were considered as constants (i.e.

heat capacity 
$$c_p = 900 \frac{J}{kg \kappa}$$
, thermal conductivity  $\lambda = 238 \frac{W}{m \kappa}$  density  $\rho = 2700 \frac{kg}{m^3}$  [55])

- The following boundary conditions were implemented at the outer edges of the model shown inFigure 9:
- 303x = 0:304 $\circ$  red line segments (i.e. inner channel walls):  $\dot{q} = \dot{q}_{wall}$ , calculated by the coupling305of the 2D and the 1D-model;306 $\circ$  Black lines (midline of the plate): adiabatic boundary condition ( $\dot{q} = 0$ ) due to307symmetry reasons;

308 • 
$$x = \frac{1}{2}(b_{plate} + b_{PCM})$$
:  $\dot{q} = 0$ , due to the fact of symmetry; and

• 
$$z = 0$$
 and  $z = H$ :  $\dot{q} = 0$ , neglecting heat losses.

310 If free convection was considered a no-slip boundary condition, the fluid flow velocity v at the
311 edges of the PCM gap was set as zero:

$$312 \bullet v = 0$$

## 313 **3.1.3** Coupling of the 2D- and the 1D-model

- The 2D- and the 1D-models were coupled to each other by the heat flux at the inner channel wall.This means:
- The volumetric heat flux  $\dot{q}_{channel}$  was calculated as an input for the 1D-model of the fluid in each time step, according to equation (4) in section 3.1.1. The mean temperature of the inner channel wall  $T_W$ , the mean temperature of the red borders in Figure 9, was calculated in the 2D-modell.
- The heat flux *q*<sub>wall</sub> was calculated in the 2D-model as an input for each inner channel wall.
   It was calculated by taking a mean value over the complete channel length of the

322volumetric heat flux  $\dot{q}_{channel}$  achieved from the 1D-model, followed by converting it into323a surface specific heat flux. As only a half of the plate was modelled, a factor if 0.5 needed324to be multiplied:

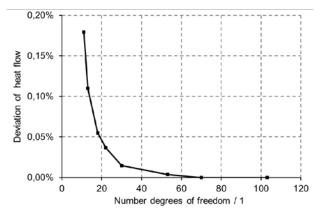
$$\dot{q}_{wall} = \frac{1}{2l} \frac{V}{A} \int_0^l \dot{q}_{channel}(y) dy$$
<sup>(14)</sup>

#### 325 3.1.4 Examination of mesh dependency

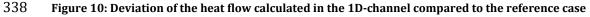
A study to determine mesh dependency was carried out. Ideally, results of a solution should be independent from the chosen number of mesh elements and time steps. In this study the aforementioned models were simulated individually to investigate the mesh dependency.

#### 329 3.1.4.1 1D-model

- To study the influence of the mesh for the 1D-model, a step temperature function  $(T_{in})$  was applied
- at the inlet of the channel. A constant temperature (the starting temperature of the step function)
- 332 was assumed at the inner channel wall  $T_W$ . Then, the simulated heat flow at the channel wall at
- the end of the simulation was compared to a reference case. Figure 10 shows the deviation of the
- heat flow in relation to the reference case with 103 degrees of freedom. The result revealed that
- a deviation lower than  $10^{-3}$  can be achieved for more than 70 degrees of freedom. Thus, a mesh
- with 70 degrees of freedom was selected.



#### 337



### 339 3.1.4.2 2D-model

340 The mesh dependency of the 2D-model mainly focused on the accuracy of free convection simula-

tion. Different from that of the 1D-model, a step temperature function was applied as the boundary condition at the channel wall. The duration of the phase change process at  $T_{PCM_{middle}}$  was in-

- vestigated as the key indicator to quantify the influence of mesh. Figure 11 presents the deviations
- between the phase change durations under different degrees of freedom compared to that of a
- reference case. It can be seen that neglectable phase change duration deviation can be achieved
- 346 when the number of degrees of freedom reached  $2.4 \times 10^5$  (see left part of Figure 11).

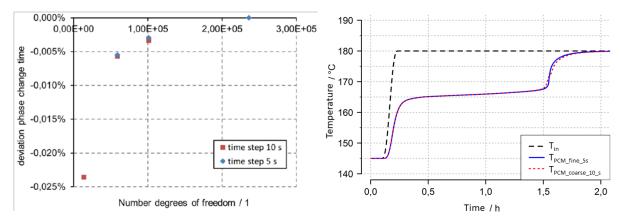


Figure 11: left: deviation of the duration of the phase change from the reference case, right: temperature forthe lowest and the highest number of degrees of freedom

349 The start and end temperatures of the phase change are determined according to DSC-measure-350 ments. Results of the mesh dependency analysis showed that the deviations of the phase change 351 duration were below 0.025 % in all cases. Increasing the time steps from 5 s to 10 s can be ne-352 glected. The right part of Figure 11 further shows the temperature at the position  $T_{PCM_{middle}}$ for 353 the reference simulation and the simulation with the lowest number of degrees of freedom. Basi-354 cally, no clear difference can be found between the two cases. Accordingly, it is believed that a 355 number of degrees of freedom of 13520 was enough to gain a sufficient accuracy at the location 356 corresponding to  $T_{PCM_{middle}}$ .

## 357 **3.1.5 Examination of free convection**

358 The melting process of a PCM is usually dominated by thermal conduction at the beginning while 359 by free convection at the end. Due to the fluid flow caused by free convection, the PCM on top 360 melts faster than the PCM at the bottom. This leads to a melting front in the shape of a S-curve. 361 [61]. This effect on the thermal behaviour of the plate storage described in section 2 was examined 362 with the assistance of the FEM-model. Therefore, a simulative comparison with and without free 363 convection was carried out for charging and discharging the storage. Table 1 summaries the ma-364 terial properties of D-mannitol used in the simulation. It is worthwhile to mention that supercool-365 ing and hysteresis between melting and solidification were neglected, and the outlet fluid temperature and the PCM-temperatures shown in Figure 9 were considered in the results. 366

367 Figure 12 shows the results for charging (left) and discharging (right). In each graph the temper-368 ature profiles for the case "with free convection" (named "conv") are marked in black, whereas 369 the temperature profiles of the case "without free convection" and thus based on conduction only 370 (named "cond") are marked in red. In both cases free convection of the PCM did not influence the outlet fluid temperature *T<sub>out</sub>* and thus the power of the storage. During the charging process, the 371 372 free convection resulted in a faster melting (around 22 min faster) of the PCM on the top of the 373 storage, compared to that at the bottom. The temperature profile of the PCM with free convection 374 is steeper than without free convection when leaving the phase change temperature after ca. 2.5 h. 375 The movement of the liquid PCM in case of free convection leads to a faster rising of the tempera-376 ture. The PCM on top crystallizes ca. 8 min slower than at the bottom during discharging. Addi-377 tionally the phase change starts 8 min later. Accordingly, the differences among the temperatures 378 at the three different heights are higher during the charging process than that during the discharg-379 ing process, due to the different phase fronts during charging and discharging [41]. It further

# verified that during discharging, natural convection can be neglected because the temperaturedifference was quite small, however, it must be considered during charging.

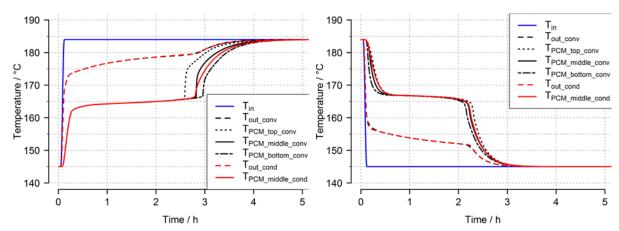


Figure 12: Simulative comparison (FEM-model) with and without free convection for the plate storage at a
 plate distance of 25 mm, left: charging, right: discharging

## 384 3.1.6 Model validation

Considering PCM degradation during the measurement, the  $c_p$ -curve of the post-test D-mannitol was used in the validation, as shown in Figure 3. The other material properties of D-mannitol were documented in Table 1. The Stefan number for this case was 0.23.

388 The mean value of the five measured inlet fluid temperatures (see Figure 4 left) was employed as

inlet fluid temperature in the simulation. The outlet fluid temperature of the measurement was

390 gained by averaging the five measured outlet fluid temperatures. The standard deviations were

391 1.13 K for the inlet fluid temperature and 0.69 K for the outlet fluid temperature, respectively.

During the measurement, a step increase temperature profile from 145 °C to 190 °C was controlled and implemented as the inlet temperature. The same step increase temperature profile was adopted in the numerical simulation as  $T_{in}$ . The mass flow rate of the whole storage was 1.5 kg/min and controlled constantly during the measurement, while in the simulation the corresponding constant mass flow rate was adopted. The measured and simulated oil outlet temperature as well as the measured and simulated PCM-temperatures at the three different locations (see Figure 9) were compared for the sake of model validation.

Heat losses of the storage were determined based on the measurements at static states before andafter the step increasing of the inlet fluid temperature. The heat loss is calculated by equation (15).

$$= \dot{m}_{oil} c_{p_{oil}} (T_{in} - T_{out})$$

(15)

- 401 The total mass flow of 1.5 kg/min and the mean value of the five measured inlet and outlet fluid 402 temperatures were utilised in the calculation of heat loss. It resulted in a heat loss of  $66 W \pm 64 W$ 403 at a temperature difference between the average exterior insulation surface and the ambient of 404 148 K. The high measurement uncertainty, which was calculated based on the *Gaussian error* 405 *propagation*, was caused by the low temperature difference between the inlet and outlet oil tem-
- 406 perature of less than 1 K in the static states.

Ò

- 407 In the model the full heat losses are considered to take place at the top and bottom edge of the 408 model at z = 0 and z = h (Figure 9).
- 409 Free convection in the liquid PCM was considered as stated in section 3.1.4.

- 410 In order to evaluate the accuracy of the simulation model, a mean deviation *D* between the meas-
- 411 ured and simulated temperatures was calculated according to Pointner et al. [32] using the fol-
- 412 lowing equation (16):

$$D = \frac{\sum_{i=0}^{i=n} |T_{sim}(t_i) - T_{meas}(t_i)|}{n+1}$$
(16)

413 where *n* is the number of measured points.

414 On the left side of Figure 13, a comparison of the temperatures between measurement and simu-415 lation is shown. Deviations between the measured and simulated outlet temperature can be ob-416 served, even though they shared a similar trend. The simulated outlet temperature strongly de-417 pended on the heat losses assumed in the model and had a relatively high measurement uncer-418 tainty. As a concequence, the simulated outlet temperature may be outside the range of the meas-419 urement uncertainty. The mean deviation between measured and simulated outlet temperature 420 (i.e. *D*out) until 5 hours was 1.15 *K*. Furthermore, the measured and simulated PCM-temperatures at the three positions given in Figure 9 were 5.65 K for  $D_{PCM_{top}}$ , 4.01 K for  $D_{PCM_{middle}}$ , and 4.18 K 421 for  $D_{PCM_{bottom}}$ , respectively. To further assess these deviations, the consideration of the tempera-422 423 ture profiles was also of necessity. It can be seen that the PCM-temperatures in measurement and 424 simulation were in parallel until 2 hours. In both measurement and simulation, the PCM at the 425 upper part melted faster than that at the lower part due to the fact of free convection (see section 426 3.1.4). There were sharp variations in the PCM-temperatures at the three measuring points from 427 2 hours to 3 hours, indicating the completion of phase transition at the corresponding locations. 428 A similarly phase change process can be seen in the simulation, however, the phase transition at 429 the three locations completed with a shorter time difference from 2.5 hours to 3 hours but the 430 total duration of the phase change is similar.

431 A further sensitivity analysis showed that the heat transfer coefficient  $\alpha$  and the locations of the 432 temperature sensors in the PCM can influence the phase change duration by approximately  $\pm 6$  %. 433 A measurement uncertainty of the PCM-density of  $\pm$  10 % influenced the duration of the phase 434 change at a similar level. Measurement uncertainty of the storage losses affected the duration of 435 the phase change up to ca. 8 %. The uncertainty of the PCM's heat capacity and melting tempera-436 ture due to its degradation additionally affected the model validation. These factors led to a high 437 uncertainty and thus the validation via the PCM-temperatures could not be carried out more ac-438 curate.

439

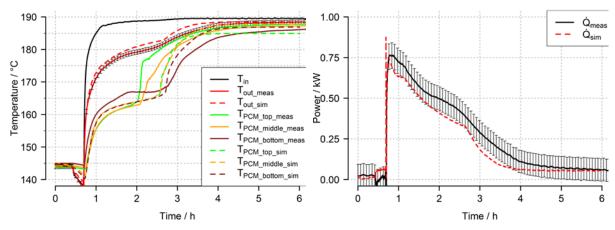


Figure 13: Validation of the FEM-model of the plate storage during charging, left: temperature profiles, right:power profile

442 On the right side in Figure 13, the profiles of the heat flow in measurement and simulation are443 shown. For both cases the heat flow was calculated according to equation (15).

444 The heat flow in the measurement was approximately 0.1 – 0.2 kW higher than in the simulation

445 until 5 hours. Thus, heat release in the measurement was higher than that in the simulation. This

446 can be explained by the described uncertainties introduced from the calculation of heat losses

447 aforementioned. From 5.5 hours on, the heat flows in measurement and simulation were identical.

448 Starting from this point of time the heat flow  $\dot{Q}$  was only influenced by the thermal losses.

## 449 3.2 Capacitance-Resistor-Models

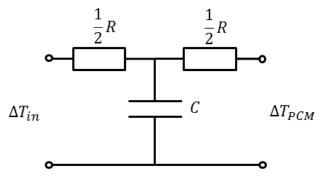
The RC-models were developed in the programming language Modelica<sup>®</sup> based on the Modelica<sup>®</sup>
 standard library version 3.2.2 [62]. Dymola Version 2017 [63] was used as modelling laboratory.

#### 452 3.2.1 Modelling of the PCM

453 To start, an element model for the PCM gap was established. Heat transfer in the PCM is based on

heat conduction or free convection, which was modelled by a thermal resistance. The storage behaviour was defined by the temperature dependent capacity of the PCM.

- 456 The PCM gap was regarded as a box-shaped element. The heat transfer was modelled in direction
- 457 of the gap width. The thermal resistance in direction of the width was divided into two thermal
- 458 resistances, between which the thermal capacitance was located. This symmetric element model
- 459 is presented in Figure 14.



460

461 Figure 14: RC-network of the PCM gap

462 The temperature dependent heat capacity C(T) was described by the enthalpy method [60] ac-

463 cording to the following equation:

$$C_{PCM}(T) = \frac{m_{PCM} \, dh(T)}{dT} \tag{17}$$

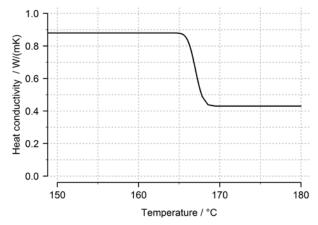
The thermal resistance of the PCM was modelled in two different ways: based on heat conductionand based on free convection (and heat conduction). These models are presented as follows:

## 466 3.2.1.1 Modelling based on heat conduction

- 467 The thermal resistance *R* based on heat conduction across a plane wall with the width of *b*, height
- 468 of *H* and thickness of *d* was determined according to equation (18) [64, S. 359]

$$R = \frac{b}{\lambda(T) H d}$$
(18)

- 469 The temperature dependent heat conductivity of D-mannitol is shown in Figure 15, considering
- 470 constant values for the liquid and the solid state, individually.



#### 471

472 Figure 15: Temperature dependent heat conductivity of D-mannitol in the simulation

## 473 3.2.1.2 Modelling based on free convection

474 If free convection during the melting process of a PCM is present, a faster melting compared to a 475 hypothetical case only based on heat conduction tends to take place. Thus, the thermal resistance 476 of the PCM is lowered. Modelling of this lowered thermal resistance was done based on correla-477 tions developed by Vogel et al. [41]. The authors introduced a so-called mean convective enhance-478 *ment factor*  $\bar{\epsilon}$ . This factor was defined by the quotient of the heat flow in a case with free convec-479 tion and an identical case without free convection for the same liquid fractions. Here, it was as-480 sumed that  $\bar{\epsilon}$  for the proposed RC-model was not only valid for the quotient of the heat flows, but 481 also for the quotient of the thermal resistances in the same cases.

482 According to Vogel et al. [41] free convection during the melting process starts at a critical liquid 483 fraction  $f_{l,crit}$ , which depends on the Rayleigh-number  $Ra_b$  calculated using the width of the gap

484 *b* and the ratio of its height and its width  $A = \frac{H}{h}$ , as described in equation (19)

$$f_{l,crit} = \sqrt[4]{\frac{150A}{Ra_b}}$$
(19)

485 The liquid fraction in the model developed in this study was calculated according to equation (20), 486 which is the ratio of the liquid mass  $m_l$  and the total PCM-mass  $m_{tot}$ :

$$f_l = \frac{m_l}{m_{tot}} \tag{20}$$

- 487 If the liquid fraction  $f_l$  was lower than the critical liquid fraction  $f_{l,crit}$  the heat transfer in the RC-
- 488 model was defined by heat conduction and the thermal resistance was calculated according to
- 489 equation (18). Otherwise, the thermal resistance was reduced by the factor  $\bar{\epsilon}$  and was calculated
- 490 according to equation (21):

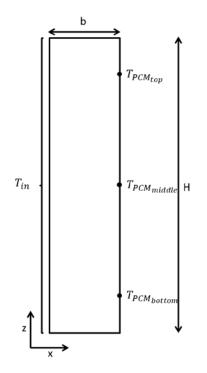
$$R_{conv} = \frac{R_{cond}}{\bar{\epsilon}}$$
(21)

491  $\bar{\epsilon}$  was calculated based on the correlation given in [41]:

$$\bar{\epsilon} = \begin{cases} 1 & f \ddot{\mathrm{u}} r \, R a_b^{\frac{1}{6}} A^{-\frac{1}{4}} < 2.73 \\ 0.57 \left( R a_b^{\frac{1}{6}} A^{-\frac{1}{4}} \right) - 0.38 & f \ddot{\mathrm{u}} r \, R a_b^{\frac{1}{6}} A^{-\frac{1}{4}} \ge 2.73 \end{cases}$$
(22)

## 492 3.2.1.3 Verification of the RC-models for PCM

- 493 The results obtained using the RC-models were compared to results obtained using the FEM-
- 494 model described in 3.1. Only the 2D-model of a PCM gap was considered for the verification. The
- 495 modelled PCM gap is shown in Figure 16.



496

497 Figure 16: Sketch of the PCM gap for comparison of the RC-model to the FEM-model

498 A step increase temperature profile was applied to the FEM-model as input at x = 0. At the other

499 three edges, the boundary condition  $\dot{q} = 0$  was implemented. At the position  $\Delta T_{in}$  of the RC-model

500 (Figure 14) the same temperature step as in the FEM-model is implemented. At the position  $\Delta T_{PCM}$ 

- the boundary condition  $\dot{q} = 0$  was applied. Identical dimension parameters and boundary condi-
- tions were applied in the RC-models as that utilized in the FEM-models.
- 503 In the following graphs the PCM-temperatures of the FEM-models are given at the three different
- heights indicated in Figure 16. However, as the RC-model cannot detail the temperature distribu-
- 505 tion over the height of the PCM, a temperature representative for a mean temperature over the
- height of the PCM gap is presented using the temperature  $T_{PCM}$  in the RC-model.

- 507 In all models the  $c_p$  respectively enthalpy curve of untreated D-mannitol shown in Figure 3 and 508 the temperature dependent heat conductivity shown in Figure 15 are adopted.
- 509 In the case based on heat conduction, the PCM-temperature over the height was constant and the
- 510 melting front was vertical, thus the temperatures at the different heights have the same profile. In
- this case, the verification was carried out by setting the gap with both width and height of 1 cm.
- 512 The RC-model was discretised in different numbers of the elements shown in Figure 14. They were 513 arranged in series in the direction of the gap width *b*. In the left picture of Figure 17, a comparison 514 between the PCM-temperatures of the RC-model and the FEM-model is shown. It is visible that a 515 finer discretization enabled the modelling result of the RC-model to gain a better agreement with 516 that of the FEM-model. The mean deviations of the PCM-temperatures between the RC and the 517 FEM-model based on equation (16) were 1.00 K for one element, 0.34 K for two elements and 518 0.21 K for three elements. The higher accuracy for a finer discretisation was reached because of 519 the temperature dependent capacity and resistance. However, a discretization of more than three 520 elements did not lead to a further relevant change of the PCM-temperature obviously. On the right 521 side of Figure 17, the thermal resistance over time in the RC-model is shown for three elements 522 and for one element. The thermal resistance increased with time due to the increase in tempera-523 ture and the temperature dependency of the heat conductivity. When all the PCM was melted, the 524 thermal resistance reached a maximum, because the PCM in completely liquid state had the lowest 525 heat conductivity. If only one element was considered, the thermal resistance incresed faster, be-526 cause of the absence of temperature gradient in the PCM modelling. This led to a slower melting 527 process and a less accurate simulation.
- 528 The RC-model for the PCM based on heat conduction was regarded as verificated when at least 2
- 529 RC elements are simulated for the given gap. It is valid for other parameters, like other gap dimen-
- 530 sions, other PCMs and other temperature steps. However, the discretisation in the direction of the
- 531 gap width must be adapted depending on the dimensions and the required accuracy of the simu-
- 532 lation model.

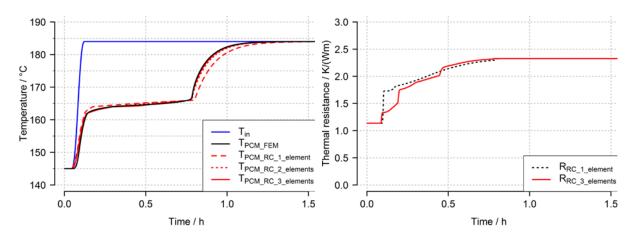


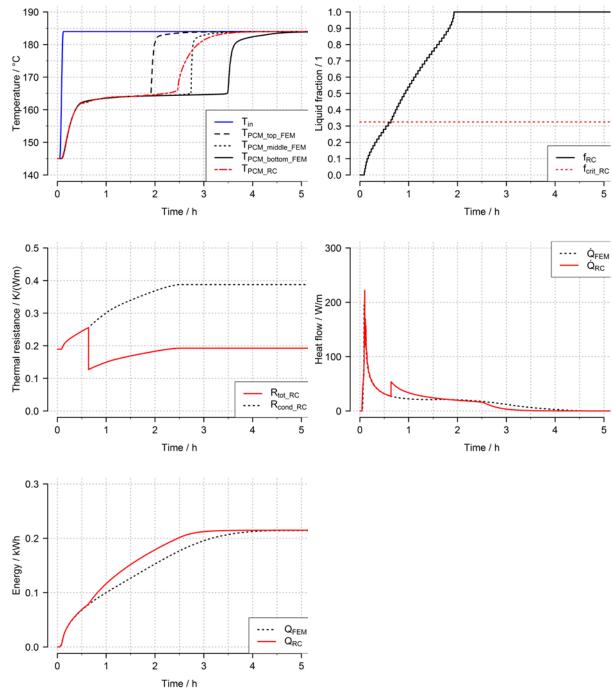
Figure 17: Comparison of the FEM-model with the RC-model of the PCM gap based on heat conduction having
 different element numbers: left: PCM-temperatures, right: thermal resistances

In case of the verification for the model with free convection the PCM gap in Figure 16, a width of25 mm and a height of 150 mm were set.

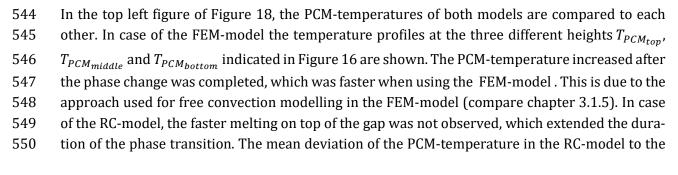
- 537 Figure 18 shows a comparison between the FEM- and the RC-model based on free convection. The
- 538 RC-model was divided into 20 RC-elements in x-direction because a higher number of elements
- does not lead to more accurate results.

540 Free convection in the FEM-model was considered based on the equations described in chapter 3.1.2.





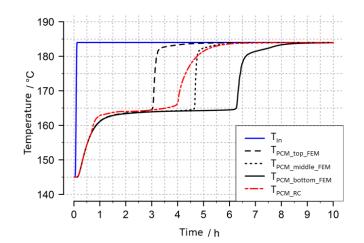
542 Figure 18: Comparison of the RC-network for the PCM gap based on free convection with the FEM-model hav-543 ing a gap width of 25 mm



- 551 PCM-temperature  $T_{PCM_{middle}}$  in the FEM-model ( $D_{PCM_{middle}}$ ) calculated according to equation (16) 552 reached 1.02 K.
- 553 In the top right graph of Figure 18, the liquid fraction as a function of time as well as the critical
- liquid fraction of the RC-model is shown. At the time t = 0.6 h the liquid fraction was higher than the critical liquid fraction which corresponded to the starting of the free convection.

556 The graph on the left side in the middle row shows the profile of the total thermal resistance in 557 the RC-model as well as the hypothetical thermal resistance only due to heat conduction. At the 558 beginning of the simulation, the total thermal resistance attributed to the thermal resistance 559 based on heat conduction. Both resistances with and without considering the free convection in-560 creased until the critical liquid fraction was reached. This was due to the continuous moving of 561 the melting front, and an increasing number of elements were melted, whose heat conductivity 562 changed from a high value in the solid phase to a low value in the liquid phase. As soon as the 563 critical liquid fraction was reached, the total thermal resistance was suddenly reduced by the fac-564 tor  $\bar{\epsilon}$  (see (21)). However, with the further moving of the melting front, the heat conductivivity 565 experienced an increasing trend again, since the number of elements under a liquid phase with 566 lower thermal conductivity kept increasing. After 2.5 hour, the whole PCM was fully melted and 567 the thermal resistance reached its final value.

- 568 In the right graphic in the middle, the heat flow is presented. After the critical liquid fraction was
- reached at 0.6 hour, the heat flow increased abruptly due the reduction of the thermal resistance.
- 570 After the fully melting of the whole PCM at 2.5 hours, the heat flow further decreased and then
- 571 stablised at a relatively low value, due to decreasing of heat transfer temperature difference. It is 572 worthwhile to mention that the transition between heat conduction and free convection in the
- 572 Worthwhile to mention that the transition between heat conduction and nee convection in the
- 573 FEM-model was smoother, and thus the start of free convection was not visible in the heat flow.
- 574 The lower left graphic shows the energy absorbed by the PCM. Due to the surge of the heat flow 575 when free convection was triggered in the RC-model, more thermal energy was absorbed at the 576 beginning than that of the FEM-model. Even this, at the end of the simulation the total energy 577 stored in both models was the same.
- 577 Stored in both models was the same.
- 578 Further verifications were carried out under different gap widths (i.e. *b*) of 10 mm to 40 mm with
- 579 steps of 5 mm. Figure 19 shows the comparison of the PCM-temperatures for both models with
- the gap width of 40 mm. The PCM-temperature in the RC-model lay between the upper and lower
- 581 PCM-temperature of the FEM-model. The mean deviation of the PCM-temperature in the RC-
- model compared to the temperature (i.e.  $D_{PCM_{middle}}$ ) in the FEM-model was 1.54 K (calculated ac-
- cording to equation (3.14)). The difference in the absorbed energy between both models was
- 584 2.5 % for a gap width of 40 mm.



585

Figure 19: Comparison of the RC-network for the PCM gap based on free convection with the FEM-model hav-ing a gap width of 40 mm

588 These results indicate that the FEM-model outperforms the RC-model in predicting the dynamic 589 behaviour of the PCM under the given boundary conditions with higher accurate. This is due to 590 the different methods utilized for simulating the free convection. In the FEM-model, the develop-591 ment of free convection within the PCM gap was considered carefully by applying the physical 592 equations described in section 3.1.2 for each individual element of the mesh. In specific, the whole 593 PCM was in solid state at the beginning, whose left part started melting first when the step in-594 crease temperature was assigned to the left border (see Figure 16). The movement of the interface 595 between molten and solid phase occurred and grew gradually towards the right part. As a conse-596 quence, the velocity of the liquid PCM started increasing slowly due to driving of the volume force. 597 In contrast, the effect of free convection in the RC-model was actived at one defined moment when 598 a certain layer thickness of molten PCM was reached, which deviated from the practical phenom-599 enon.

600 The above discussion in terms of accuracy for the RC-model and the FEM-model under different

601 PCM gaps with free convection is considered to be sufficient to reveal the model validity. Accord-

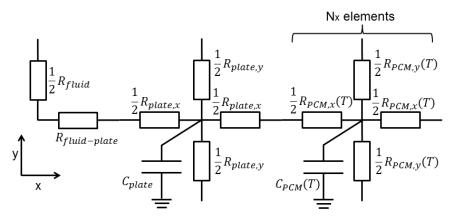
ingly, the model is valid under the given boundary conditions and a maximal examined gap width

603 of 40 mm.

Since in most applications the simulation of the whole storage is more interesting than only simulating one PCM gap, a RC-model for the whole storage including the plates is introduced in the
next section.

## 607 3.2.2 Plate heat exchanger

The plate storage was modelled by a network of thermal resistances and capacitances. It coupled the above-described RC-model for the PCM, a similar RC-model for the plate, and a thermal resistance for heat transfer modelling in the liquid side. In x-direction, the same section as marked in Figure 6 was modelled, thus half a plate and half a PCM gap. In y-direction, i.e. in direction of the fluid flow, a thermal resistance for the heat transfer in the fluid was modelled. The model assumptions number 2, 4, 6, 7, 8 and 9 described in section 3.1 were considered. The RC-network is shown in Figure 20.





616 Figure 20: RC-network of a basic cell of the plate storage

617 On the right side the RC-model for the PCM described in chapter 3.2.1 is part of the RC-network of 618 the plate storage. The PCM-model can be divided into an arbitrary number of elements in x-direc-619 tion (i.e. in direction of the gap width), followed by a similar RC-circuit for the heat transfer in 620 direction of the width of the plate which consisted of two resistances and a capacity. The re-621 sistances of the plate  $R_{plate}$  were calculated according to equation (18) with constant material 622 properties. The capacity of the plate ( $C_{plate}$ ) was also kept as a constant, which was calculated 623 according to equation (23) [64, S. 71]:

$$C = m c_p \tag{23}$$

624 The thermal resistance due to the heat transfer between fluid and plate ( $R_{fluid-plate}$ ) was calcu-625 lated based on equation (24)

$$R_{fluid-plate} = \frac{1}{\alpha A} \tag{24}$$

The heat transfer coefficient *α* was calculated by a Nusselt-correlation according to [57, S. 151].
The area *A* is the surface area between the plate and the fluid.

628 The thermal resistance  $(R_{fluid})$  was calculated according to equation (25).

$$R_{fluid} = \frac{1}{\dot{m} c_{p_{fluid}}} \tag{25}$$

To simulate the behaviour of one total plate thereby calculating the variation of the fluid temperature in the flow direction, the RC-network was repeated by *n* times in flow direction. The thermal resistances of the single RC-networks were connected to each other in flow direction. Thus, the heat transfer in y-direction included by that from the fluid flow in the channel (the thermal resistance  $R_{fluid}$ ) and by that from the heat conduction across the plate ( $R_{plate,y}$ ) and the PCM ( $R_{PCM,y}$ ).

#### 635 3.2.2.1 Verification

To verificate the RC-plate model, a comparison to the FEM-model was implemented. The comparison was done for both simulations with and without free convection. In the case without free convection, the RC-model was compared to a 3D-FEM-model without free convection. The plate distance and the mass flow per plate were set as 25 mm and 0.3 kg/min, respectively. The RCmodel was divided into 16 elements in the flow direction and into 8 elements in direction of the width of the PCM gap. In the case with considering free convection, a FEM-model with free convection with the 2D-1D-coupling described in sections 3.1.1 to 3.1.3 was utilized as a benchmark 643 for the sake of comparisoin, and the plate distance and the mass flow rate were set as 50 mm and644 0.3 kg/min, respectively.

- In Figure 21 the results of both comparisons are shown, in which the left side illustrates the case
- 646 without free convection while the comparison cases with free convection are presented on the
- right side. The outlet fluid temperature and the PCM-temperature of both models had almost iden-
- tical profile in the comparison when free convection was not considered. The mean deviation of
- 649 the outlet fluid temperatures ( $D_{out}$ ) based on equation (16) was 0.15 *K* and the mean deviation 650 of the PCM-temperature ( $D_{PCM}$ ) reached 0.28 *K*. It indicates that under the given boundary con-
- 651 ditions, the RC-plate model without free convection can simulate the storage with a comparable
- accuracy to that of the 3D-FEM-model.
- In the case with free convection, differences in the profile of the outlet fluid temperature and the
- 654 PCM-temperature can be clearly observed. It is due to the different modelling approaches of the
- 655 RC-model and the FEM-model with free convection described in section 3.2.1.3. After 2 hours, free
- 656 convection started in the RC-model, thereby leading to a decrease in thermal resistance and the
- outlet fluid temperature. However, in the FEM-model, this transition occured smoothly and no
- significant variation in the outlet fluid temperature can be seen. The mean deviation between the
- 659 RC- and the FEM-model ( $D_{out}$ ) based on equation (16) was 0.62 K. The PCM-temperature was
- similar in both models until when the phase change was completed. In the RC-model, the phase
- change completed approximately half an hour earlier than in the FEM-model at the middle PCM
- 662 position corresponding to  $T_{PCM_{middle}}$ . This difference was approximately 8 % of the whole phase
- change duration of 6 hours. The mean deviation of the PCM-temperature between the RC-model
- and the PCM-temperature ( $D_{PCM}$ ) in the FEM-model reached 0.85 K.

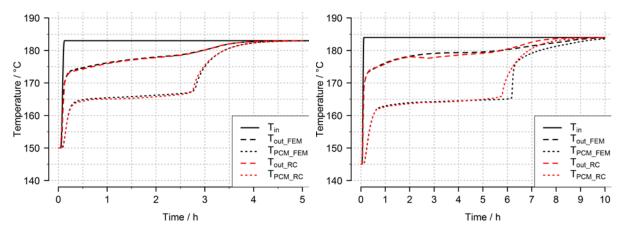


Figure 21: Comparison between the RC- and the FEM-model for the plate strorage: left: without free
 convection in the PCM, right: with free convection in the PCM

667 The results demonstrate that the accuracy of the RC-models under the present boundary conditions is high enough to be used as dimensioning tool during charging, in both cases without 668 669 and with considering free convection. The model without considering free convection can be valid 670 for other dimensions and material properties, but the discretization of the model has to be 671 adapted in order to reach the required accuracy. In case of the model with considering free 672 convection, the validity was carried out under the described boundary conditions and dimensions, 673 and the given examined gap width up to 50 mm. In this case the discretization must also be 674 adapted to reach results in the desired accuracy range. It should be examined in future, if the 675 model with free convection is still valid for boundary conditions outside this range.

## 676 4 Conclusion and outlook

677 In this study a PCM storage based on FracTherm®-plates for process heat applications is pre-678 sented. To the best of the authors knowledge, it is the first time that these plates were used as heat 679 exchangers in PCM-storages. The PCM in the storage was filled between the gaps created by these 680 plates in the PCM storage. A simplified RC-model for the storage was developed and assessed, and 681 it was found that free convection during the melting process influences the heat transfer and must 682 be considered in such kind model. This is a novelty which is missing in the published research 683 from the literature. Thus, an innovative RC-model considering free convection of the PCM during 684 the melting process in a rectangular gap was developed, validated and presented in this study. In 685 addition, a FEM-model based on a simplified approach considering free convection was presented 686 and validated for the sake of comparison.

- The FEM-model was validated using experimental data from a lab-scale test rig. The outlet fluid temperature simulated was consistent with that of the experiment, with a mean deviation of 1.15 K. The RC-model for the rectangular PCM gap can predict the temperature at half height of the FEM-model with a maximal mean deviation of 1.54 K compared to the FEM-model. The results verify that the RC-model for the PCM gap with free convection was reliable and can be applied under the maximal examined gap width up to 40 mm. However, the RC-model including free convection is not valid for the solidification process of the PCM. A comparison of both models for the
- 694 whole plate storage presented a mean deviation of the outlet fluid temperatures of 0.62 K and a
- 695 mean deviation of the PCM-temperature of 0.85 K. The simulation time of the phase changing pro-
- 696 cess using the RC-model was 20 to 30 times faster than that using the FEM-model.
- It is believed that the RC-model developed in this study was a useful and computation-effectivetool for model-based PCM storage design and optimization, while taking into account free convec-
- tion of the PCM during melting. Despite its simplicity, it can still provide sufficient accuracy com-
- parable to that of the FEM-model. The RC-model developed can also be easily adapted for the sim-
- valuation of other latent heat storages where the PCM is contained in rectangular gaps and heated
- 702 from one side.
- 703 It should be examined in future to see whether the model is still valid for a higher gap widths and
- gap height than the here examined dimensions and whether it is valid for other PCMs. In addi-
- tional, it maybe valuable to further improve the RC-model by considering the free convection dur-
- 706 ing the solidification process for different PCMs.
- For both models presented, i.e. the simplified RC-model and the more detailed FEM-model, they
  can be utilized to properly size the PCM-plate-storages by determining the acceptable parameter
  ranges under the given application requirements. Furthermore, a more in-depth design optimiza-
- tion of PCM storages can be followed and detailed using the FEM-model.

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