Searching for Rotational Symmetries Based on the Gestalt Algebra Operation $\prod$

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***Abstract*— Among the Gestalt algebra operations the one for rotational symmetries needs a special search strategy. This contribution briefly recalls Gestalt algebra in overview and then concentrates on the rotational "mandalas". Though these are very salient to human perception, they have not been studied much in the classical over hundred years old Gestalt literature. A search procedure is given which can find such patterns in sets of primitive Gestalten extracted from images. Experiments are performed using selected pictures from the 2013 symmetry recognition competition data and SIFT key points as primitive extractor.**

Keywords—Gestalt perception; combinatorial search; rotational symmetry; visual saliancy;

I. Gestalt Algebra

Gestalt Algebra has been introduced as an attempt to capture recursive hierarchies of symmetric patterns in a mathematical setting [1]. Such patterns are obviously salient to the human visual system and probably meaningful for visual recognition tasks such diverse as foreground/background discrimination or automatic understanding of building structure from aerial images of urban terrain. Gestalt Algebra is defined on the Gestalt domain G containing 2D position *po*, scale *sc*, orientation *or*, rotational frequency *fr*, and assessment *as*.

$$G = \mathbb{R}^2 \times (0,\infty) \times (\mathbb{R}/\mathbb{Z}) \times \mathbb{N} \times [0,1]$$

Each element g of the domain is called a Gestalt. Figure 1 below shows some example Gestalten displayed on screen. Position and scale are intuitively evident. Assessment is indicated by grey-tone – black=1 being good and white=0 being meaningless (and invisible). Frequency *fr(g)* is indicated as number of spokes of the wheel shape. Thus rotation by *2π/fr(g)* does not change the identity. The orientation attribute gives the angle between the x-axis and the first spoke counterclockwise – value 1 being the maximal angle *2π/fr(g)*. In [1] two operations | and ∑ are defined on the domain one binary and one *n*-ary:

$$|: G^2 \to G: \quad h = g_1 \,|\, g_2$$
$$\Sigma: G^n \to G: \quad h = \sum g_1 ... g_n$$

These are defined for any pair or *n*-tupel of Gestalten respectively, but will yield high assessment values only for Gestalten pairs arranged in mirror-symmetry for |, and rows of Gestalten arranged in good continuation and similar spacing along a row for ∑. Detailed definitions and proofs of algebraic closure can be found in [1] or [2].

II. Related Work

Related work can be found in textbooks such as [3]. But the topic is being studied for more than a hundred years, with [4] being one of the most known elder examples. In [5] a very interesting approach to a recursive hierarchy for visual objects is given, where emphasis is on assessing the instances (not called Gestalten there) by the minimum description length criterion. Unfortunately, this has not been further pursued. Based on statistical a-contrario tests Gestalt perception is thoroughly investigated in [6]. In that work there are hints on the recursive nature of Gestalten but all examples remain one-step deep. A quite successful example of the published state of rotational symmetry recognition is given with [7]. This approach is based on correlating large numbers of image parts of many scales with other such image parts. It is suitable for mirror- and rotational-symmetries. It is a rather typical example of listing all possible such mappings (discretizing mirror-axes or centers, and scales) and assessing each such hypothesis top-down. Instead the approach given in this contribution constructs a hierarchy of possibilities bottom-up, starting with primitives extracted at interest-point locations in scale-space.

III. Rotational Symmetries

For Rotational Symmetries in [2] a third operation ∏ is defined on the Gestalt domain and a proof sketch is given for algebraic closure concerning this operation. Like ∑ it is *n*-ary:

$$\Pi: G^n \to G: \quad h = \prod g_1 ... g_n$$

∏ prefers rotational arrangements with incrementally rising orientations as shown in Fig. 1 below. Fig. 1 also shows an interface meant for testing and understanding Gestalt algebra operations. As mentioned above assessment is displayed as grey tone (black is one = perfect, white is zero = meaningless); the rotational aggregate Gestalt constructed here by operation ∏ of the three smaller parts is not perfect, because the parts are not of equal size, do not perfectly fit into an equilateral triangle, and their orientations do not perfectly increment in 2/3π steps. Thus it appears in a grey tone. It would be even lighter e.g. if the parts were much further apart (or too close together), or if the orientations would deviate stronger. While there are closed form solutions for | and ∑, the operation ∏ requires an initialization and Newton iteration. For lack of

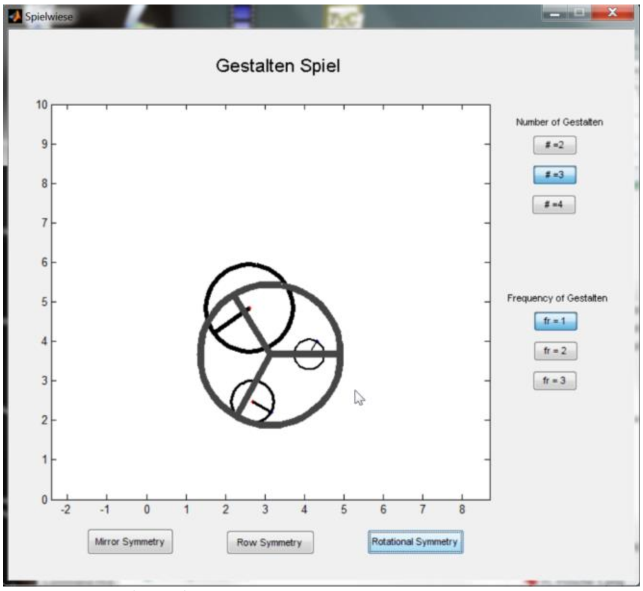

Fig. 1. Interface for interactive Gestalt algebra operations: Three frequency one Gestalten (parts) give a resulting aggregate Gestalt of frequency three

space the technical details – such as the Jacobean used for the iteration – are omitted here with reference to [2].

IV. Search

Searching for Gestalten with high assessment, with k given primitives from an image, is a task that may demand high computational efforts. Listing all pairs for tests with the mirror operation | is obviously of order $\mathcal{O}(k^2)$. But already for one step using the n-ary row operation $\sum$ brute-force exhaustive search would be of order $\mathcal{O}(2^k)$. And that is only for one step. Searching recursively several steps deep for Gestalten with high assessment is more demanding. For practical work up to now heuristics have been used [8] [9]. E.g. for $\sum$ search starts with pairs and tries to prolong the row by appending Gestalten to both ends until the assessment of the aggregate row Gestalt starts dropping. In the following a new rationale for searching for $\prod$ Gestalten is given:

As with $\sum$ search starts with pairs. Preliminarily, only the case $fr(g)=fr(h)=1$ will be handled (e.g. primitive Gestalten). For these the following steps are performed:

- For each pair $(g,h)\epsilon G$ the difference of orientations is investigated in the domain of radians: $d=2\pi(or(h)\text{-}or(g))$. In case $d>\pi$ (h,g) will be further investigated instead of (g,h). Given a minimal assessment $1\text{-}\epsilon$ for the aggregate Gestalt a number of possible arities $n_1,\ldots,n_m$ is suggested. E.g. for the pair of Gestalten in Fig. 2 $d=2/5\pi$ suggesting arities $n_1=4$, $n_2=5$, and $n_3=6$. A global upper threshold is set for n_m in case d is too small.
- For each arity n_i a center results, indicated as starting point of the polygon in Figure 2. The next two vertices are $po(g)$ and $po(h)$. For $n_i>2$ The following $n_i\text{-}2$ vertices give the positions where queries for partner Gestalten are constructed.

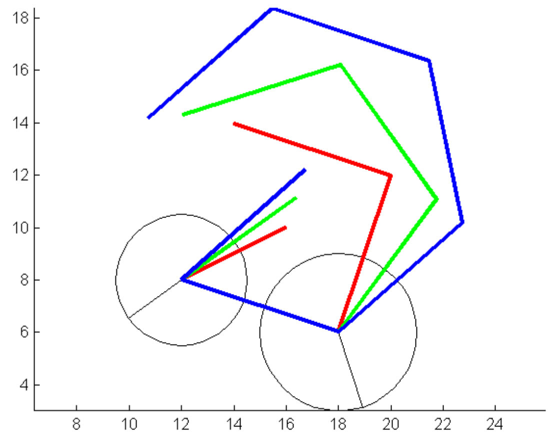

Figure 2: Given a pair (g,h) three orbits are suggested leading to corresponding queries

These queries demand in conjunctive combination: 1) position close to the vertex, 2) orientation in accordance with the orbit run index, and 3) scale compatible with the geometric mean $(sc(g)sc(h))^{0.5}$. Again the minimal assessment $1\text{-}\varepsilon$ for the aggregate Gestalt determines the thresholds for the queries. For each of the vertices $2<j\leq n_i$ the Gestalt f_j is chosen that best fits according to three criteria.

- If the query for one of the vertices j returns an empty set no aggregate will be constructed. Else $\prod ghf_3\ldots f_{ni}$ is calculated by the mentioned iterative method. This is a greedy search: at most one Gestalt per search region is found. Thus, and because the arities are bounded by n_m, the complexity remains polynomial, namely $\mathcal{O}(k^3)$.

Note that arity 2 is also possible. For this case no search is required. For the case $fr(g)=fr(h)>1$ the search is more complicated. More regular polygons have to be constructed according to the inherent symmetry of such parts.

V. Experiments

Experiments are made with some selected pictures from the Penn State symmetry recognition data [10]. SIFT key-points are used for the primitives and only to these the search procedure outlined above is applied. Figure 3a shows some example image; and 3b displays the 422 primitive Gestalten of frequency one that are obtained from this image using the standard SIFT key-point extractor with default parameter settings. Figure 4a displays the 41 rotational Gestalten that result from the search procedure outlined above using ε=0.1. In Figure 4b the centers of these Gestalten are overlaid in blue color to a lighter version of the picture.

As usual a cluster analysis has to follow the search. One Gestalt alone is often illusory, but an image structure salient to human observers will often cause a dominant cluster of Gestalten. With rotational Gestalten most of the members found have too low frequency, because only some sub-set of

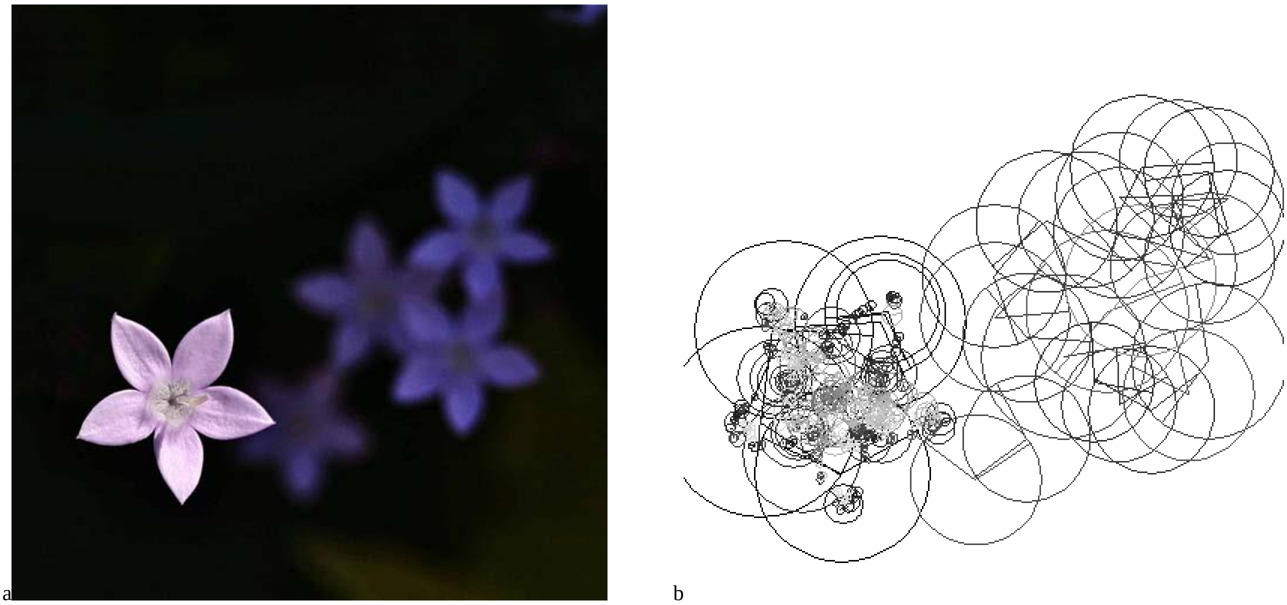

Fig. 3: Example from the 2013 CVPR competition data: a. original image, b. primitive Gestalten

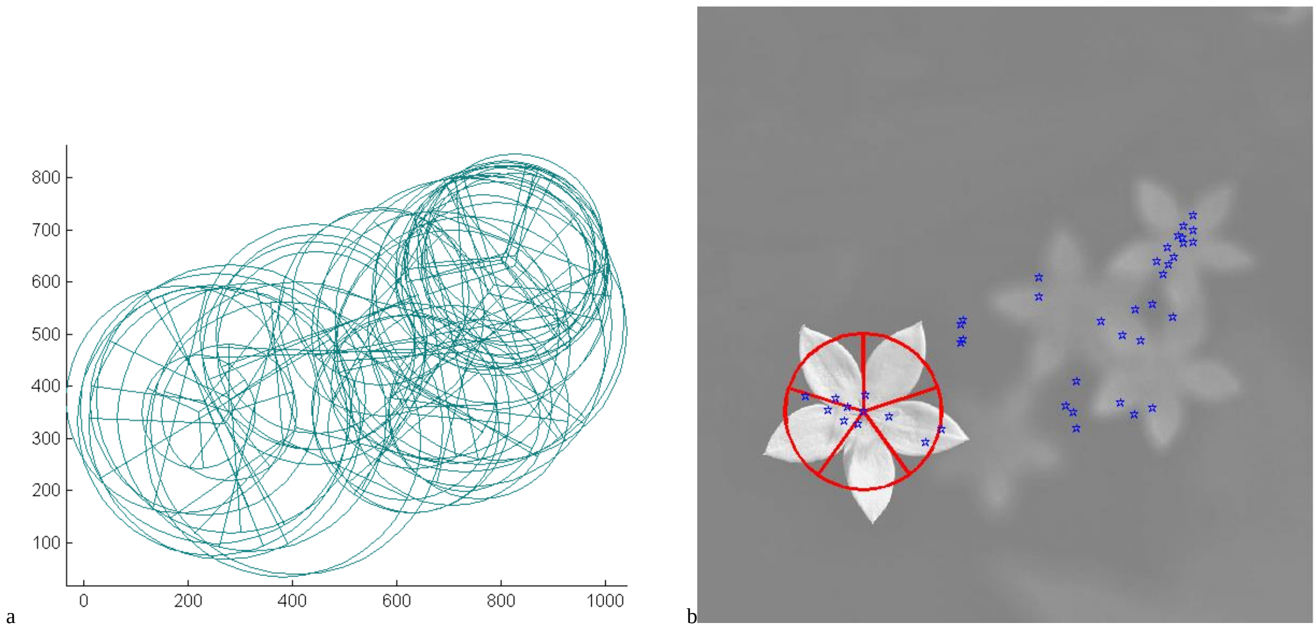

Fig. 4: Result on the data shown in Figure 3

the correct parts is grouped, or a sub-group in the algebraic sense is established (for non-prime frequencies). Thus we trust most in the maximal frequency found within each cluster. Clustering is seeded by the maximally assessed Gestalt. In the example the corresponding cluster is on the salient flower to the lower left and contains 11 Gestalten. Highest frequency in that cluster is five, and the corresponding Gestalt (draw in red color) almost perfectly fits human perception. After removing this cluster the next best seed Gestalt is to the lower right. It is illusory and draws only few neighbors. The third cluster is located close to the less salient flower in the upper right. It contains only frequencies up to three and is displaced a little. Results on others of the easier images of the set are similar, but most of the images in this collection are too difficult for the Gestalt algebra approach in its current state.

VI. Conclusion

It may be stated that it is possible to recognize rotational symmetries from natural images using SIFT key-points as primitive Gestalten, the operation $\prod$ as underlying mathematical model, and the indicated new search strategy. However, as already the easy-looking example displayed in Fig. 3 sows, very often some of the n parts of a rotational

Gestalt may not manifest as appropriate SIFT-key-points. Carefully looking at Fig. 3b one may notice, that the rightmost primitive of the salient pentagon is actually missing. With the other four parts being arranged perfectly, the search is forced to insert some less well fitting Gestalt in the role of this missing part. The resulting correct Gestalt of frequency five is thus too small in scale and not the best assessed in its cluster. Obviously, strategies will be required for hallucinating missing parts in order to achieve more impressing recognition performance rates.

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