Stability of global climate cooperation under uncertainty

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Abstract

International cooperation is needed to substantially reduce global greenhouse gas emissions and avoid dangerous climate change. The possibility of cooperation is influenced by the presence of uncertainty in both damages from climate change and the development of low-carbon technologies. This paper integrates uncertainty into an analysis of the stability of global climate cooperation, using cooperative game theory. I find that the deterministic result does not necessarily carry over to the case including uncertainty, and that the stability of global cooperation crucially depends on the ability of a coalition to redistribute risk between members with different levels of risk aversion. The results suggest that risk redistribution should feature prominently in the international climate regime.

Keywords: climate; cooperative game; core stability; uncertainty

JEL: C71, D81, H41, Q20, Q54

1 Introduction

Global greenhouse gas emissions need to be substantially reduced over the coming decades in order to avoid dangerous climate change. As emission reductions in one country benefit all other countries, international negotiations are needed to incentivize emission reductions. This process has led to the adoption of the Paris Agreement at the 21st Conference of the Parties (COP21), a framework under which emission reduction pledges are submitted voluntarily by each country.

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The stability of international cooperation on emission reductions has been the topic of many game-theoretic analyses. The standard model based on the *internal and external stability* concept finds that meaningful cooperation is only stable among few countries (e.g. Barrett, 1994; Carraro and Siniscalco, 1993; Diamantoudi and Sartzetakis, 2006). However, stability among a larger group of countries can be achieved by including trade restrictions or R&D in the agreement, or by modifying some assumptions of the original model (see Hovi et al., 2015, for an overview of extensions and modifications of the model). In contrast, analyses based on the *core stability* concept find global cooperation to be stable (Chander and Tulkens, 1997; Helm, 2001). This concept is based on a unanimity rule, i.e. cooperation can only occur on a global level and if all countries agree to cooperate. Such a structure aligns well with the setup of the international climate negotiations, where "consensus" is needed to adopt decisions.

In the past years, several game-theoretic models based on internal and external stability have been expanded to include uncertainty about one or multiple parameter values. Bramoullé and Treich (2009) find that uncertainty about the impact of a global pollutant can have a positive environmental effect, as risk averse polluters reduce their emissions. However, Benchekroun and van Long (2013) find that this result does not hold up in a dynamic game with a stock pollutant. Barrett (2013) analyzes a threshold game and finds that uncertainty about the threshold causes cooperation to be unstable.

A few studies have included uncertainty about mitigation costs. Hong and Karp (2014) use a theoretical model with uncertainty in the mitigation cost parameter and find that higher risk aversion of countries increases membership in a stable climate agreement, but reduces mitigation levels. Kolstad and Ulph (2011) and Finus and Pintassilgo (2013) use a similar theoretical model with uncertainty in the benefit-cost ratio of mitigation. They find that such uncertainty can increase the size of a stable coalition, if countries are symmetric under uncertainty, but asymmetric after the true nature of the parameter is revealed. However, this negative effect of the removal of uncertainty can mostly be avoided by a suitable transfer scheme (Dellink and Finus, 2012). Dellink et al. (2008) employ a numerical model and find that uncertainty in the mitigation benefit parameters has a larger influence on the stability of coalitions than uncertainty in the mitigation cost parameters.

To the best of my knowledge, no attempt has been made to introduce uncertainty into a model of international climate cooperation using core stability. This is despite the good alignment of the concept with the international climate negotiations and the striking result of the deterministic model. However, the underlying concept of a cooperative game has been extended into a *cooperative game with stochastic payoffs* (Suijs et al., 1999;

Timmer et al., 2005), including modified definitions of preferences, the objective function of coalitions, allocations, and of the core. Two distinct allocation concepts are put forward by Suijs et al. (1999) and Timmer et al. (2005).

This paper provides the first inclusion of uncertainty in a model of international climate cooperation using core stability. In particular, this paper determines whether the result of stable global cooperation in the deterministic model carries over into a model with uncertainty. The game is extended into a cooperative game with stochastic payoffs, based on adapted preferences and objective functions. I analyze two model setups, one including technological uncertainty and one including uncertainty in climate damages. Further, I check the (non-)emptiness of the core for both allocation concepts put forward in the literature, for both types of uncertainty.

Section 2 gives a general overview of the model and the different game-theoretic concepts. Section 3 analyzes the model with technological uncertainty, while Section 4 analyzes uncertainty in climate damages. Section 5 situates the results in the general context of international climate negotiations and policy. Section 6 concludes.

2 Uncertainty in a model of climate cooperation

The model in this paper is based on the deterministic model of transfrontier pollution by Chander and Tulkens (1997), hereafter CT model. Let $N = \{1, .., n\}$ be the set of players, representing countries, of the cooperative game. Then each player i is characterized by a production function $P_i(E_i)$ and a damage function $D_i(E_N)$. The production function depends on the player's own emissions E_i and describes the costs of reducing emissions. The damage function describes the costs of climate change and depends on global emissions $E_N = \sum_{i \in N} E_i$. The difference of production and damage function gives the utility of player i:

$$u_i(E_i, E_N) = P_i(E_i) - D_i(E_N). \tag{2.1}$$

In the subsequent sections of this paper, either the production or the damage function are modified to include uncertainty. They become a *stochastic variable*. Consequently, the utility of player i also becomes a stochastic variable U_i:

$$U_i(E_i, E_N) = P_i(E_i) - D_i(E_N). \tag{2.2}$$

In order to define the *cooperative game with stochastic payoffs*, some adjustments to the deterministic model are needed, following Suijs et al. (1999). As payoffs are now stochastic, a *preference ordering* $\succeq_i$ is needed, which indicates whether player i prefers one stochastic payoff to another. This paper uses the specific preference ordering proposed by Suijs et al. (1999). Let $\alpha_i \in (0, 0.5]$ and $q_{\alpha_i}(X)$ be the α_i-quantile of payoff X. Then player i prefers payoff X to payoff Y if and only if the α_i-quantile of payoff X is larger than the α_i-quantile of payoff Y:

$$X \succeq_i Y :\Leftrightarrow q_{\alpha_i}(X) \geq q_{\alpha_i}(Y). \tag{2.3}$$

This means that $q_{\alpha_i}(X)$ is the *certainty equivalent* of payoff X, $ce_i(X)$. For payoffs with symmetric distribution[1], $\alpha_i = 0.5$ implies risk neutral behaviour and $\alpha_i < 0.5$ implies risk averse behaviour, i.e.

$$ce_i(X) < \mathbb{E}(X). \tag{2.4}$$

I will occasionally refer to the *level of risk aversion*, which is higher with lower α_i[2]. Note that $\alpha_i > 0.5$, i.e. risk loving behaviour, is excluded, for reasons that will become apparent later in the paper.

To determine emissions and value for each possible coalition $S \subseteq N$, I follow the original CT model. Let $j \notin S$. Then player j maximizes (the certainty equivalent of) its individual utility:

$$\max_{E_j} ce_j(U_j(E_j, E_N)). \tag{2.5}$$

All players $i \in S$ maximize the sum of (the certainty equivalent of) the utility of all members:

$$\max_{(E_i)_{i \in S}} \sum_{i \in S} ce_i(U_i(E_i, E_N)). \tag{2.6}$$

These parallel optimizations provide deterministic emission levels for all players. The *value function* $V(S)$ provides the payoffs for each coalition $S \subseteq N$. It is given by the sum of the utility of all members of the coalition, and is therefore a stochastic variable:

$$V(S) = \sum_{i \in S} U_i(E_i, E_N). \tag{2.7}$$

2.1 Allocation concepts

In order to check whether a stable global agreement is possible, I use the concept of the *core*. For the deterministic case, the core contains all allocations of the value of the grand coalition N, such that no coalition can

[1] This will be the case throughout the paper.

[2] The level of risk aversion as used in this paper should not be confused with the *degree of risk aversion*, which is defined as the ratio of second and first derivative of an agent's utility function (Pratt, 1964; Arrow, 1965).

achieve a higher payoff by defecting from the grand coalition. This requires the definition of the concept of an *allocation*. For games with stochastic payoffs, two different concepts of an allocation exist: the concept of *pure payoff allocation* by Timmer et al. (2005) and the concept of *expectation and risk allocation* by Suijs et al. (1999).

The concept of pure payoff allocation defines an allocation of $V(S)$ as a vector of multiples of $V(S)$ (Timmer et al., 2005). Let $g \in \mathbb{R}^{|S|}$ with $\sum_{i \in S} g_i = 1$. Then player $i \in S$ receives $g_i V(S)$.

In the concept of expectation and risk allocation (Suijs et al., 1999), an allocation is defined by two vectors, one for the expected value of the stochastic payoff, and one for the risk. Let $(d, r) \in \mathbb{R}^{|S|} \times \mathbb{R}_+^{|S|}$ with

$$\sum_{i \in S} d_i = \mathbb{E}(V(S)), \tag{2.8a}$$

$$\sum_{i \in S} r_i = 1. \tag{2.8b}$$

Then player $i \in S$ receives

$$d_i + r_i(V(S) - \mathbb{E}(V(S))). \tag{2.9}$$

The pure payoff allocation concept is a direct translation from the deterministic allocation concept, as every player receives a share of the coalition value. However, as the coalition value is now stochastic, this means that a player can only receive a payoff if she also takes on risk. For risk averse players, the certainty equivalent of the received payoff is reduced by this presence of risk. The expectation and risk allocation concept remedies this drawback by allowing for an independent allocation of the expected value and the risk of the stochastic payoff. As a consequence, it is possible for very risk averse players to offload (some of) their risk on to less risk averse players, improving (the certainty equivalent of) total utility. On the other hand, risk neutral players can "insure" other players against risk by taking it on themselves, and receive some compensation for this service.

For both allocation concepts, an allocation X of $V(N)$ is in the core of the game, if for no coalition $S \subset N$ there exists an allocation Y of $V(S)$, such that $Y_i \succ X_i$ for all players $i \in S$. In other words, all members of a coalition S need to agree on a deviation from the grand coalition N.

For the concept of pure payoff allocation, several convexity concepts exist, which all imply that the core of the game is not empty (Timmer et al., 2005). For the concept of expectation and risk allocation, the core of the game is not empty if and only if (Suijs et al., 1999)

$$\max_{i \in N} q_{\alpha_i}(V(N)) \geq \sum_{S \subset N} \delta_S \max_{i \in S} q_{\alpha_i}(V(S)), \tag{2.10}$$

where δ_S is a *balanced map*, i.e.

$$\sum_{S \subset N} \delta_S \mathbb{1}_{i \in S} = 1 \; \forall i \in N. \tag{2.11}$$

In the following sections, I will apply both allocation concepts to stochastic models of climate cooperation. Section 3 considers technological uncertainty, while Section 4 considers uncertainty in climate damages.

3 Technological uncertainty

This section considers uncertainty in mitigation costs, i.e. in the production function. For the mean μ, I use a quadratic function in emission reductions. Uncertainty increases with emission reductions, as more new technologies need to be applied. The distribution is set, such that the standard deviation, σ, increases quadratically with emission reductions, in line with the deterministic mitigation costs. A normal distribution is used. Specifically, the model is

$$P_i(E_i) \sim \mathcal{N}\left(\underbrace{P_i^0 - \gamma_i(E_i^0 - E_i)^2}_{\mu}, \underbrace{(\theta(E_i^0 - E_i)^2)^2}_{\sigma^2}\right), \tag{3.1}$$

with

- P_i^0: baseline production level
- E_i^0: baseline emission level
- γ_i: mitigation cost parameter
- θ: amount of uncertainty parameter
- μ: mean of normal distribution
- σ^2: variance of normal distribution

For $\alpha \leq 0.5$, this formulation ensures that, in terms of production, a higher emission level is preferred to a lower emission level, i.e. emission reductions are costly[3].

[3]For $\alpha > 0.5$, this would not necessarily be the case, as the higher risk resulting from emission reductions is preferred to lower risk. Therefore, this case is excluded throughout the paper.

Damages from climate change are assumed to be known with certainty in this section. They increase quadratically with global emissions E_N:

$$D_i(E_N) = \pi_i E_N^2, \tag{3.2}$$

where π_i is the damage cost parameter.

Taking production and damage together, utility is given by

$$\begin{aligned} U_i(E_i, E_N) &= P_i(E_i) - D_i(E_N) \\ &\sim \mathcal{N}\left(P_i^0 - \gamma_i(E_i^0 - E_i)^2 - \pi_i E_N^2, (\theta(E_i^0 - E_i)^2)^2\right). \end{aligned} \tag{3.3}$$

As utility is normally distributed, the certainty equivalent of utility is

$$\begin{aligned} ce_i(U_i(E_i, E_N)) &= \mu + \sigma z(\alpha_i) \\ &= P_i^0 - \gamma_i(E_i^0 - E_i)^2 - \pi_i E_N^2 + \theta(E_i^0 - E_i)^2 z(\alpha_i), \end{aligned} \tag{3.4}$$

where $z(\alpha_i)$ is the *probit function*

$$z(\alpha_i) = \sqrt{2}\ erf^{-1}(2\alpha_i - 1), \tag{3.5}$$

$$erf(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2}\ dt. \tag{3.6}$$

$erf(x)$ is the so-called *error function.* $z(\alpha_i)$ is strictly increasing and $z(0.5) = 0$. Consequently, $z(\alpha_i) \leq 0$ for the range of α_i considered in this paper.

Players determine their emission levels by optimizing the certainty equivalent of utility. Proposition 1 shows the resulting emission levels and the impact of uncertainty.

Proposition 1. *Let $S \subset N$. For the game with technological uncertainty,*

(i) emission levels of individual players ($E_i(S)$) and global emissions ($E_N(S)$) are given by

$$E_N(S) = \frac{E_N^0}{\pi_S \sum_{i\in S} \frac{1}{\gamma_i - \theta z(\alpha_i)} + \sum_{j\notin S} \frac{\pi_j}{\gamma_j - \theta z(\alpha_j)} + 1}, \tag{3.7a}$$

$$E_i(S) = E_i^0 - \frac{\pi_S}{\gamma_i - \theta z(\alpha_i)} E_N(S) \quad \forall i \in S, \tag{3.7b}$$

$$E_j(S) = E_j^0 - \frac{\pi_j}{\gamma_j - \theta z(\alpha_j)} E_N(S) \quad \forall j \notin S, \tag{3.7c}$$

with $\pi_S = \sum_{i\in S} \pi_i$.

(ii) increases in the amount of uncertainty (θ) or in the level of risk aversion ($-z(\alpha)$) influence emission levels like an increase of the mitigation cost (γ).

(iii) increases in the amount of uncertainty (θ) or in the level of risk aversion ($-z(\alpha)$) lead to higher emission levels.

For reasons of readability, all theoretical proofs can be found in Appendix A.

Proposition 1 (ii) and (iii) follow intuitively from the fact that all players are risk averse or risk neutral. As emission reductions cause higher uncertainty, an additional "cost" of mitigation is created. This cost is treated similarly to the deterministic mitigation cost γ. As the damage function is unchanged compared to the deterministic case, the benefit from emission reductions is also unchanged. Consequently, the optimal emission levels under technological uncertainty are higher than without uncertainty.

3.1 Stability of global cooperation under pure payoff allocation

I now consider the stability of global cooperation in the game with technological uncertainty. Specifically, I check whether the core of the cooperative game with stochastic payoffs is non-empty for all cases, as in the deterministic case. First, I consider the game under pure payoff allocation. Proposition 2 shows that the result from the deterministic case cannot be transferred to the uncertain setup.

Proposition 2. *Under pure payoff allocation, the core of the cooperative game with stochastic payoffs and technological uncertainty can be empty.*

The proof is given by the following simple counter-example.

Example 1. *Let $N = \{1, .., 4\}$ and*

$$\alpha = \begin{bmatrix} 0.001 \\ 0.1 \\ 0.5 \\ 0.5 \end{bmatrix}, \quad \gamma = \pi = E^0 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \quad P^0 = \begin{bmatrix} 10 \\ 10 \\ 10 \\ 10 \end{bmatrix}. \tag{3.8}$$

The resulting game is given in Table 4 in Appendix B.

The four players in Example 1 only differ in their risk preferences. Player 1 is very risk averse, player 2 is modestly risk averse, while players 3 and 4 are risk neutral. As a result of their different levels of risk aversion, the certainty equivalent of a payoff differs between the players. Figure 1 shows the relation between the share of $V(N)$ a player receives and the respective certainty equivalent of the payoff, for all players.

A high share of $V(N)$ increases the payoff in expectation, but also increases the level of risk. We see that player 1 actually prefers a payoff of zero

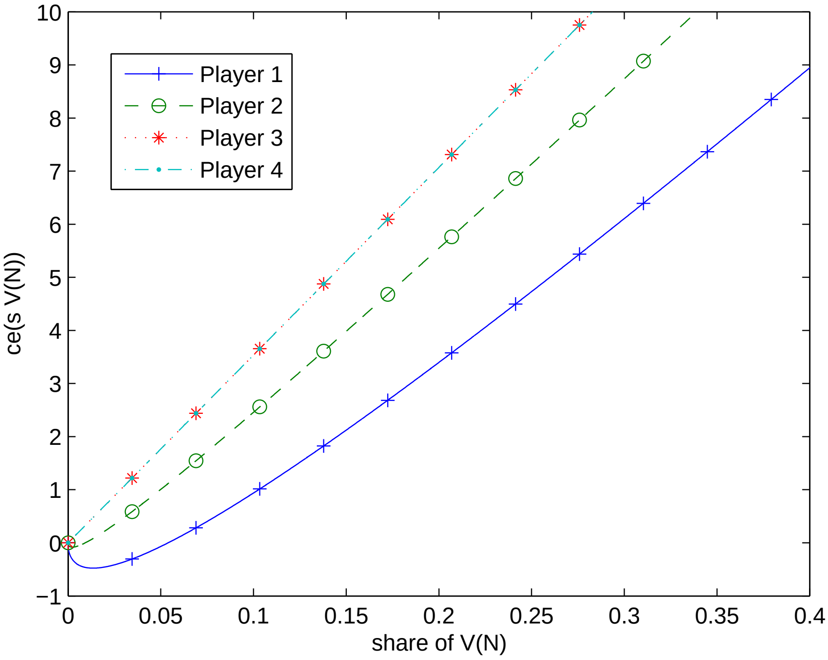

Figure 1: Certainty equivalent of payoff for varying share of $V(N)$, for all players in Example 1.

to a small share of $V(N)$, due to its high level of risk aversion. For a share larger than roughly 0.05, the increase in expectation has the larger effect and player 1 prefers the higher share. Still, the larger risk for higher shares of $V(N)$ causes the certainty equivalent of the payoff to increase at the smallest rate of all players. Player 2 experiences the same phenomenon, but the impact of risk is much smaller. For players 3 and 4, who are risk neutral, only the expected value of their share of $V(N)$ matters, and consequently the certainty equivalent of their payoff increases linearly with the share of $V(N)$.

To now show that the core of this game is empty, I compare the payoff of the grand coalition N to the payoffs of all singleton coalitions $\{i\}$. Specifically, I determine the minimum share of $V(N)$ each player needs to equal its payoff in the singleton case. For example, player 1 compares its payoff to the payoff of singleton coalition $\{1\}$. The corresponding certainty equivalent is 8.5319. In Figure 1, one can see that player 1 would need a share of 0.3857 of $V(N)$ or higher to prefer a proposed global allocation to the singleton payoff.
This calculation can be done for all players. The certainty equivalent of the singleton payoffs and the resulting minimum shares of $V(N)$ are shown in

Table 1.

	Player 1	Player 2	Player 3	Player 4
Singleton payoff	8.5319	8.3033	7.6406	7.6406
Minimum share	0.3857	0.2865	0.2162	0.2162

Table 1: Singleton payoffs and minimum shares of $V(N)$ to satisfy corresponding singleton coalition for all players in Example 1.

The sum of the minimum shares of all players is 1.1046. As this is larger than one, no global allocation can satisfy all singleton coalitions simultaneously. Therefore, the core of the game is empty. The result is driven by the risk averse players 1 and 2. For them to join the grand coalition, both need a share of $V(N)$ substantially larger than 0.25 to compensate for the technological uncertainty associated with reaching low emission levels. While the risk neutral players 3 and 4 would be content with less than a uniform allocation, they cannot lower their share enough to allow sufficient redistribution to the risk averse players.

The result is also driven by the heterogeneity of risk aversion between the four players. If all players were as risk averse as player 1, the emission level in the grand coalition would remain higher, reducing technological uncertainty. Consequently, each player would be content with a share of $V(N)$ of 0.25 and the core of the game would not be empty. Similarly, if all players were risk neutral, the game would be similar to the deterministic case and therefore the core would not be empty. Overall, the combination of very risk averse players with risk neutral players causes the emptiness of the core in Example 1, as the wishes of both groups of players cannot be sufficiently reconciled in the grand coalition.

3.2 Stability of global cooperation under expectation and risk allocation

The previous section showed that under pure payoff allocation, the core of the game can be empty. This is caused by the inability for very risk averse players to receive an appropriate payoff without also taking on risk. The expectation and risk allocation concept in this section allows independent allocation of expectation and risk. This reinstates the result of core non-emptiness from the deterministic case. For the analytical proof, two simplifying assumptions are needed.

Assumption 1. *(i) All players are symmetric in all parameters except α_i, the level of risk aversion.*

(ii) At least one player is risk-neutral, i.e. $\alpha_i = 0.5$.

Proposition 3. *Under expectation and risk allocation and Assumption 1, the core of the cooperative game with stochastic payoffs and technological uncertainty is non-empty.*

Numerical simulations suggest that Proposition 3 also holds when Assumption 1 is not fulfilled. No counter-example was found.

Intuitively, Proposition 3 is a direct result from the possibility of independent risk allocation. This allows the least risk averse player to take on all risk in the grand coalition. If this player is risk neutral, the game is similar to the deterministic case and the deterministic core allocation is also in the core of the stochastic game. If the least risk averse player is not risk neutral, the acceptance of risk by this player in the grand coalition still creates a total utility surplus, compared to smaller coalitions. Most of this utility surplus can then be allocated to the least risk averse player as compensation for the acceptance of risk, so that this player also agrees to global cooperation. As the other players are made better off by being allowed to offload risk, this mechanism allows for a core stable allocation.

4 Uncertainty in climate damages

This section considers uncertainty in the damage function. In a reversal of the setup in the last section, the production function is known with certainty,

$$P_i(E_i) = P_i^0 - \gamma_i(E_i^0 - E_i)^2, \tag{4.1}$$

while the damage function is normally distributed

$$D_i(E_N) \sim \mathcal{N}\left(\pi_i E_N^2, (\kappa_i E_N^2)^2\right), \tag{4.2}$$

with parameter κ_i for the amount of uncertainty. Similar to the last section, $\alpha_i \leq 0.5$ ensures that, when considering damages, a lower global emission level is preferred to a higher global emission level, i.e. emission reductions reduce climate damages.

Utility is given by

$$U_i(E_i, E_N) = P_i(E_i) - D_i(E_N) \sim \mathcal{N}\left(P_i^0 - \gamma_i(E_i^0 - E_i)^2 - \pi_i E_N^2, (\kappa_i E_N^2)^2\right) \tag{4.3}$$

with certainty equivalent

$$ce_i(U_i(E_i, E_N)) = P_i^0 - \gamma_i(E_i^0 - E_i)^2 - \pi_i E_N^2 + \kappa_i E_N^2 z(\alpha_i). \tag{4.4}$$

Proposition 4 shows the emission levels resulting from the optimization of the certainty equivalents of utility and the effect of uncertainty.

Proposition 4. *Let $S \subset N$. For the game with uncertainty in climate damages,*

(i) emission levels of individual players and global emissions are given by

$$E_N(S) = \frac{E_N^0}{(\pi_S - (\kappa z(\alpha))_S) \sum_{i \in S} \frac{1}{\gamma_i} + \sum_{j \notin S} \frac{\pi_j - \kappa_j z(\alpha_j)}{\gamma_j} + 1}, \tag{4.5a}$$

$$E_i(S) = E_i^0 - \frac{\pi_S - (\kappa z(\alpha))_S}{\gamma_i} E_N(S), \quad \forall i \in S \tag{4.5b}$$

$$E_j(S) = E_j^0 - \frac{\pi_j - \kappa_j z(\alpha_j)}{\gamma_j} E_N(S) \quad \forall j \notin S, \tag{4.5c}$$

with $\pi_S = \sum_{i \in S} \pi_i$ and $(\kappa z(\alpha))_S = \sum_{i \in S} \kappa_i z(\alpha_i)$.

(ii) increases in the amount of uncertainty (κ) or in the level of risk aversion ($-z(\alpha)$) influence emission levels like an increase of the damage cost (π).

(iii) increases in the amount of uncertainty (κ) or in the level of risk aversion ($-z(\alpha)$) lead to lower emission levels.

Similar to Section 3, Proposition 4 (ii) and (iii) follow directly from the risk aversion of the players. As uncertainty is tied to the global emission level, emission reductions reduce uncertainty in this setup. Therefore, an extra benefit of mitigation is created by the inclusion of uncertainty, which is added to the deterministic benefit of mitigation, π. Consequently, emission levels are lower than without uncertainty.

4.1 Stability of cooperation under pure payoff allocation

Again, I first consider the (non-)emptiness of the core of the cooperative game with stochastic payoffs under pure payoff allocation. In contrast to the game with technological uncertainty, global cooperation in the game with uncertainty in climate damages reduces total risk compared to the singletons case, as global emission levels are lowest under global cooperation. This can be seen in the standard deviation (σ), which increases quadratically with global emissions. Therefore, the impossibility of risk redistribution under the pure payoff allocation concept is not as detrimental to stable global cooperation as under technological uncertainty, because less risk needs to be redistributed. Consequently, the core of the game is non-empty for simple games similar to Example 1. However, games with empty core still exist, as Proposition 5 shows.

Proposition 5. *Under pure payoff allocation, the core of the cooperative game with stochastic payoffs and uncertainty in climate damages can be empty.*

The proof is given by the following counter-example.

Example 2. *Let* $N = \{1, 2, 3\}$ *and*

$$\alpha = \begin{bmatrix} 0.001 \\ 0.1 \\ 0.5 \end{bmatrix}, \quad \kappa = \begin{bmatrix} 0.01 \\ 0.1 \\ 0.5 \end{bmatrix}, \quad \gamma = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \quad \pi = \begin{bmatrix} 0.1 \\ 0.01 \\ 0.001 \end{bmatrix}$$

$$P^0 = 10^4 \begin{bmatrix} 1 \\ 5 \\ 5 \end{bmatrix}, \quad E^0 = 10^2 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}. \tag{4.6}$$

The resulting game is given in Table 5 in Appendix B.

The three players in Example 2 differ in their risk preferences, with player 1 being very risk averse, player 2 being modestly risk averse and player 3 being risk neutral. They also differ in their personal amount of risk (κ) and in the deterministic damage cost (π). Player 1 is very risk averse, but experiences relatively little risk, while having the highest deterministic damage cost. Player 2's amount of risk and deterministic damage cost are both modestly high, similar to the level of risk aversion. Player 3 is risk neutral, while his amount of risk is the highest of all players and the deterministic damage cost is the lowest. Figure 2 again shows the relation between the share of $V(N)$ a player receives and the payoff.

The picture is similar to the technological uncertainty case. Player 1 needs a share of $V(N)$ of roughly 0.2 or larger in order prefer the payoff to zero, due to the detrimental impact of risk. Player 2 experiences the same mechanism, but the effect is much smaller, while for player 3 the payoff increases linearly with the share of $V(N)$.

	Player 1	Player 2	Player 3
Singleton payoff	0.17E+04	4.12E+04	4.99E+04
Minimum share	0.2170	0.5390	0.4948

Table 2: Singleton payoffs and minimum shares of $V(N)$ to satisfy corresponding singleton coalition for all players in Example 2.

For the determination of core emptiness, the minimum shares of $V(N)$ to satisfy the singleton coalition of each player can be calculated, similar to Example 1. The results are shown in Table 2 and the sum of all minimum shares is 1.2508. Consequently, the core of the game is empty. In contrast to Example 1, the result is not driven by the most risk averse player, who actually requires less than a uniform allocation share, as the singleton payoff is relatively low. This is due to the low baseline production level P^0, which

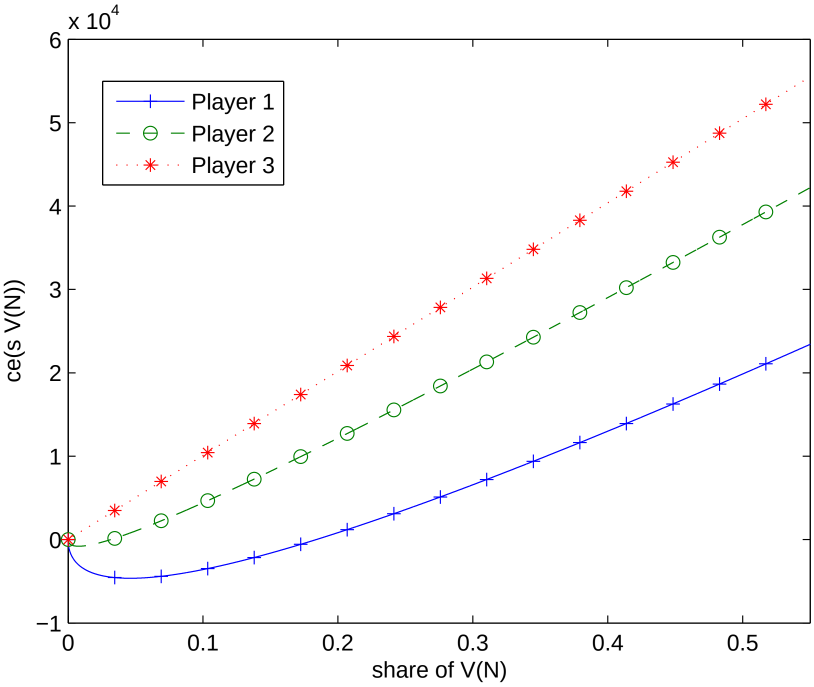

Figure 2: Certainty equivalent of payoff for varying share of $V(N)$, for all players in Example 2.

for player 1 is substantially lower than for players 2 and 3. A high share of $V(N)$ is rather required by player 2, because the modest level of risk aversion still means that the payoff growth in Figure 2 is substantially less than for the risk neutral player 3. Further, the singleton payoff of player 2 is relatively high, because the baseline production level is among the highest of all players. Consequently, a very high share of $V(N)$ is needed to compensate player 2 for the effect of risk.

Again, heterogeneity between the players is required for the result of core emptiness. If the players were symmetric in all parameters, the chosen emission level in the grand coalition, and its associated level of risk, would conform with the wishes of all players equally. Then, a uniform allocation of $V(N)$ to all players would be stable and the core of the game would not be empty. Asymmetry in the parameters can concentrate the detrimental effects of risk on one or a few players and thereby cause an empty core.

4.2 Stability of cooperation under expectation and risk allocation

Finally, I check whether the expectation and risk allocation concept reinstates the result of core non-emptiness from the deterministic case, as it did under technological uncertainty. This is indeed the case, as Proposition 6 shows.

Proposition 6. *Under expectation and risk allocation, the core of the cooperative game with stochastic payoffs and uncertainty in climate damages is non-empty.*

The intuitive reasoning behind the result is similar to the technological uncertainty setup, as the independence of risk allocation allows the least risk averse player to take on all risk in the grand coalition. This creates a total utility surplus, which can be used to compensate the least risk averse player for the acceptance of risk, making all players better off under global cooperation.

5 Discussion

This section situates the theoretical results in the general context of international climate policy. Uncertainty continues to play a large role in the climate policy discourse, as the economic effects of climate change are still quite uncertain and many current estimates do not account for some factors likely to influence the impact of climate change, such as changing weather patterns or tipping points (Burke et al., 2015; IPCC, 2014). Further, the future development of low-carbon technologies, and their costs, remain highly uncertain. For example, carbon capture and storage usually plays a prominent role in long-term scenarios compatible with ambitious emission reduction targets, but faces several technical, economic and political uncertainties (Watson et al., 2014; Koelbl et al., 2014). On the other hand, some low-carbon technologies have surprised with rapid cost reductions, such as the 80% drop in the costs of photovoltaic modules in mature markets between 2008 and 2012 (IEA, 2014).

In this context, the results of this paper lend themselves to two conclusions. First, the results suggest that global emission levels react differently to technological uncertainty and uncertainty in climate damages, in a situation with risk averse actors. While technological uncertainty introduces an additional mitigation cost, the presence of uncertainty in climate damages introduces an additional mitigation benefit. Consequently, global emission levels are higher under technological uncertainty, and lower under uncertainty in climate damages, compared to the situation without uncertainty. This suggests that research on low-carbon technologies, reducing mitigation costs and technological uncertainty, could be particularly effective in lowering global GHG emissions. On the other hand, some amount of uncertainty in climate damages helps to reduce emission levels, as risk averse actors react to uncertainty by increasing their mitigation efforts. While this result is also supported by the analysis of Bramoullé and Treich (2009), based on internal and external stability, Benchekroun and van Long (2013) find that it no longer holds up in a dynamic game with a stock pollutant. Further, in the actual climate policy discourse, uncertainty about the effects of climate change is often used to justify a delay of mitigation efforts, notably by US President George W. Bush in his announcement that he would not support the Kyoto Protocol (White House, 2001). Supporters of the precautionary principle, on the other hand, argue that high climate uncertainty should compel ambitious mitigation efforts (e.g. Grant and Quiggin, 2014; Lewandowsky et al., 2014).

Second, the results on core stability in this paper suggest that the ability of risk redistribution between countries is a key determinant for stability of a global climate agreement. For both technological uncertainty and uncertainty in climate damages, global cooperation can be unstable under pure payoff allocation, where risk cannot be detached from the payoff. However, if the expectation and risk allocation concept is used, where risk can be freely distributed, a stable global agreement always exists.

This suggests that risk redistribution should be incorporated in the international climate regime. In particular, stable cooperation might be aided by the willingness of some countries to shoulder some of the risk of other countries, in addition to their own. Arguably, developed countries are better equipped to deal with with unexpected events than developing countries, due to higher wealth and well established governmental structures. Therefore, developed countries could shoulder additional risk, in exchange for more ambitious emission reductions from developing countries. This would both reduce the global impact of climate uncertainty and harmonize international emission reduction efforts.

The redistribution of risk might be achieved via the Warsaw International Mechanism on Loss and Damage (WIM, e.g. UNFCCC, 2013), which was established at the 19th Conference of the Parties (COP19) in 2013 and intends to address climate loss and damage in developing countries. The importance of loss and damage was further recognized in the Paris Agreement, which includes an independent article devoted to the subject (UNFCCC, 2015a, art. 8). While the WIM currently focuses on knowledge and dialogue, it was requested at COP21 "to establish a clearing house for risk transfer" (UNFCCC, 2015b, para. 48). The mandate of the WIM could further be enhanced at a scheduled review at COP22.

6 Conclusions

This paper integrates technological uncertainty and uncertainty in climate damages into the model of international climate negotiations using core stability, by Chander and Tulkens (1997). This requires adjustments of several concepts of cooperative game theory, most prominently the concept of an allocation, for which two distinct concepts are put forward in the literature. I find that the deterministic result of core non-emptiness does not necessarily carry over to the uncertain setup, if the pure payoff allocation concept is used. However, the expectation and risk allocation concept reinstates the deterministic result, by allowing independent allocations of expected value of a payoff and of its risk. The possibility of risk redistribution, for example through the Warsaw International Mechanism on Loss and Damage, might therefore improve the opportunity for global agreement in the international climate negotiations.

The model in this paper could be extended or modified in a few ways. First, the shape of the production and the damage functions could be generalized or changed. Second, other probability distributions and preference orderings could be used. Third, technological uncertainty and uncertainty under climate damages could both be considered in a joint model, or other types of uncertainty could be introduced.

A Proofs

A.1 Proof of Proposition 1

Proof. Let $S \subset N, i \in S, j \notin S$. Then j optimizes the certainty equivalent of individual utility. $E_j(S)$ and $E_N(S)$ denote emissions of player j and global emissions, respectively, given that coalition S has formed.

$$\begin{aligned} ce_j(U_j(E_j(S), E_N(S))) &= P_j^0 - \gamma_j(E_j^0 - E_j(S))^2 - \pi_j E_N(S)^2 + \theta(E_j^0 - E_j(S))^2 z(\alpha_j) \\ 0 \stackrel{!}{=} \frac{\partial ce_j(U_j(E_j(S), E_N(S)))}{\partial E_j(S)} &= 2\gamma_j(E_j^0 - E_j(S)) - 2\pi_j E_N(S) + 2\theta z(\alpha_j)(E_j^0 - E_j(S)) \\ \Rightarrow (\gamma_j - \theta z(\alpha_j))(E_j^0 - E_j(S)) &= \pi_j E_N(S) \\ \Rightarrow E_j(S) &= E_j^0 - \frac{\pi_j}{\gamma_j - \theta z(\alpha_j)} E_N(S) \end{aligned}$$

Player i optimizes the sum of all certainty equivalents of utility of the members of S:

$$\sum_{l \in S} ce_l(U_l(E_l(S), E_N(S))) = \sum_{l \in S} P_l^0 - \gamma_l(E_l^0 - E_l(S))^2 - \pi_l E_N(S)^2 + \theta(E_l^0 - E_l(S))^2 z(\alpha_l) \tag{A.1}$$

$$\begin{aligned} 0 &\stackrel{!}{=} \frac{\partial \sum_{l \in S} ce_l(P_l(E_l(S)) - D_l(E_N(S)))}{\partial E_i(S)} \\ &= 2\gamma_i(E_i^0 - E_i(S)) - 2\underbrace{\sum_{l \in S} \pi_l}_{=:\pi_S} E_N(S) - 2\theta z(\alpha_i)(E_i^0 - E_i(S)) \end{aligned}$$

$$\begin{aligned} \Rightarrow \pi_S E_N(S) &= (\gamma_i - \theta z(\alpha_i))(E_i^0 - E_i(S)) \\ \Rightarrow E_i(S) &= E_i^0 - \frac{\pi_S}{\gamma_i - \theta z(\alpha_i)} E_N(S) \end{aligned}$$

The sum of individual emissions of all players gives global emissions $E_N(S)$:

$$\begin{aligned} E_N(S) &= \sum_{l \in N} E_l(S) = \sum_{i \in S} E_i(S) + \sum_{j \notin S} E_j(S) \\ &= \sum_{i \in S} E_i^0 - \frac{\pi_S}{\gamma_i - \theta z(\alpha_i)} E_N(S) + \sum_{j \notin S} E_j^0 - \frac{\pi_j}{\gamma_j - \theta z(\alpha_j)} E_N(S) \\ &= E_N^0 - \underbrace{\left(\pi_S \sum_{i \in S} \frac{1}{\gamma_i - \theta z(\alpha_i)} + \sum_{j \notin S} \frac{\pi_j}{\gamma_j - \theta z(\alpha_j)} \right)}_{=:\chi(S)} E_N(S) \end{aligned}$$

$$\Rightarrow E_N(S) = \frac{E_N^0}{\chi(S) + 1}$$

$\chi(S)$ is the reduction factor of global emissions, showing Proposition 1 (i). The second and third parts of the Proposition follows directly from (i).

□

A.2 Proof of Proposition 3

Proof. As mentioned in Section 2.1, the core of the game is not empty if and only if condition (2.10) is fulfilled.

In our setup, the maximum quantile of a payoff is

$$\max_{i\in S} q_{\alpha_i}(V(S)) = \max_{i\in S} \mu(S) + \sigma(S)z(\alpha_i) = \mu(S) + \sigma(S)\max_{i\in S} z(\alpha_i) \tag{A.2}$$

$$= \mu(S) + \sigma(S)z\left(\max_{i\in S}\alpha_i\right), \tag{A.3}$$

as z is monotonically increasing in α. Assuming at least one risk-neutral player, this leads to

$$z\left(\max_{i\in N}\alpha_i\right) = z(0.5) = 0 \tag{A.4}$$

and

$$z\left(\max_{i\in S}\alpha_i\right) \leq 0 \quad \forall S \subset N. \tag{A.5}$$

As $\sigma(S) \geq 0$, condition 2.10 is implied by

$$\begin{aligned}
&\mu(N) \geq \sum_{S\subset N} \delta_S\ \mu(S) \\
&\Leftrightarrow \quad P_N^0 - \pi_N\left(\pi_N \sum_{i\in N}\frac{\gamma_i}{(\gamma_i - \theta z(\alpha_i))^2} + 1\right)E_N(N)^2 \\
&\quad \geq \sum_{S\subset N}\delta_S P_S^0 - \pi_S\left(\pi_S\sum_{i\in S}\frac{\gamma_i}{(\gamma_i-\theta z(\alpha_i))^2}+1\right)E_N(S)^2
\end{aligned} \tag{A.6}$$

Global emissions under different coalitions are related by

$$\begin{aligned}
E_N(S) &= \frac{E_N^0}{\chi(S)+1} = \frac{E_N^0}{(\chi(N)+1)\frac{\chi(S)+1}{\chi(N)+1}} \\
&= E_N(N)\frac{\chi(N)+1}{\chi(S)+1}
\end{aligned} \tag{A.7}$$

Using (A.7), condition (A.6) becomes

$$
\begin{aligned}
&-\pi_N\left(\pi_N\sum_{i\in N}\frac{\gamma_i}{(\gamma_i-\theta z(\alpha_i))^2}+1\right)E_N(N)^2\\
&\geq -\sum_{S\subset N}\delta_S\pi_S\left(\pi_S\sum_{i\in S}\frac{\gamma_i}{(\gamma_i-\theta z(\alpha_i))^2}+1\right)\left(E_N(N)\frac{\chi(N)+1}{\chi(S)+1}\right)^2\\
\Leftrightarrow\quad &\pi_N\left(\pi_N\sum_{i\in N}\frac{\gamma_i}{(\gamma_i-\theta z(\alpha_i))^2}+1\right)\\
&\leq \sum_{S\subset N}\delta_S\pi_S\left(\pi_S\sum_{i\in S}\frac{\gamma_i}{(\gamma_i-\theta z(\alpha_i))^2}+1\right)\left(\frac{\chi(N)+1}{\chi(S)+1}\right)^2\\
\Leftrightarrow\quad &\pi_N\left(\pi_N\sum_{i\in N}\frac{\gamma_i}{(\gamma_i-\theta z(\alpha_i))^2}+1\right)\\
&\leq \sum_{S\subset N}\delta_S\pi_S\left(\pi_S\sum_{i\in S}\frac{\gamma_i}{(\gamma_i-\theta z(\alpha_i))^2}+1\right)\left(\frac{\pi_N\sum_{i\in N}\frac{1}{\gamma_i-\theta z(\alpha_i)}+1}{\pi_S\sum_{i\in S}\frac{1}{\gamma_i-\theta z(\alpha_i)}+\sum_{j\notin S}\frac{\pi_j}{\gamma_j-\theta z(\alpha_j)}+1}\right)^2
\end{aligned}
$$

Using symmetry, we have

$$
\begin{aligned}
\Leftrightarrow\quad &n\pi\left(n\pi\sum_{i\in N}\frac{\gamma}{(\gamma-\theta z(\alpha_i))^2}+1\right)\\
&\leq \sum_{S\subset N}\delta_S s\pi\left(s\pi\sum_{i\in S}\frac{\gamma}{(\gamma-\theta z(\alpha_i))^2}+1\right)\left(\frac{n\pi\sum_{i\in N}\frac{1}{\gamma-\theta z(\alpha_i)}+1}{s\pi\sum_{i\in S}\frac{1}{\gamma-\theta z(\alpha_i)}+\sum_{j\notin S}\frac{\pi}{\gamma-\theta z(\alpha_j)}+1}\right)^2\\
&=\pi\left(n\pi\sum_{i\in N}\frac{\gamma}{(\gamma-\theta z(\alpha_i))^2}+1\right)\\
&\quad\sum_{S\subset N}\delta_S s\frac{\left(s\pi\sum_{i\in S}\frac{\gamma}{(\gamma-\theta z(\alpha_i))^2}+1\right)}{\left(n\pi\sum_{i\in N}\frac{\gamma}{(\gamma-\theta z(\alpha_i))^2}+1\right)}\left(\frac{n\pi\sum_{i\in N}\frac{1}{\gamma-\theta z(\alpha_i)}+1}{s\pi\sum_{i\in S}\frac{1}{\gamma-\theta z(\alpha_i)}+\sum_{j\notin S}\frac{\pi}{\gamma-\theta z(\alpha_j)}+1}\right)^2
\end{aligned}
$$

Define

$$
A:=\left(s\pi\sum_{i\in S}\frac{\gamma}{(\gamma-\theta z(\alpha_i))^2}+1\right)\left(n\pi\sum_{i\in N}\frac{1}{\gamma-\theta z(\alpha_i)}+1\right)^2, \tag{A.8a}
$$

$$
B:=\left(n\pi\sum_{i\in N}\frac{\gamma}{(\gamma-\theta z(\alpha_i))^2}+1\right)\left(s\pi\sum_{i\in S}\frac{1}{\gamma-\theta z(\alpha_i)}+\sum_{j\notin S}\frac{\pi}{\gamma-\theta z(\alpha_j)}+1\right)^2. \tag{A.8b}
$$

In the next step, I show that

$$
\sum_{S\subset N}\delta_S s\frac{A}{B}\geq n, \tag{A.9}
$$

completing the proof.

For $S = N$, we immediately have $A = B$. Let $S \subsetneq N$. Then

$$\begin{aligned}
A =& \left(s\pi \sum_{i\in S} \frac{\gamma}{(\gamma - \theta z(\alpha_i))^2} + 1 \right) \left(n\pi \sum_{i\in N} \frac{1}{\gamma - \theta z(\alpha_i)} + 1 \right)^2 \\
=& \pi^3 \left[sn^2 \left(\sum_{i\in N} \frac{1}{\gamma - \theta z(\alpha_i)} \right)^2 \left(\sum_{i\in S} \frac{\gamma}{(\gamma - \theta z(\alpha_i))^2} \right) \right] \\
&+ \pi^2 \left[2sn \left(\sum_{i\in N} \frac{1}{\gamma - \theta z(\alpha_i)} \right) \left(\sum_{i\in S} \frac{\gamma}{(\gamma - \theta z(\alpha_i))^2} \right) + n^2 \left(\sum_{i\in N} \frac{1}{\gamma - \theta z(\alpha_i)} \right)^2 \right] \\
&+ \pi \left[s \left(\sum_{i\in S} \frac{\gamma}{(\gamma - \theta z(\alpha_i))^2} \right) + 2n \left(\sum_{i\in N} \frac{1}{\gamma - \theta z(\alpha_i)} \right) \right] \\
&+ 1 \\
=&: a_3\pi^3 + a_2\pi^2 + a_1\pi + 1,
\end{aligned}$$

and

$$\begin{aligned}
B =& \quad \left(n\pi \sum_{i\in N} \frac{\gamma}{(\gamma - \theta z(\alpha_i))^2} + 1 \right) \left(s\pi \sum_{i\in S} \frac{1}{\gamma - \theta z(\alpha_i)} + \pi \sum_{j\notin S} \frac{1}{\gamma - \theta z(\alpha_j)} + 1 \right)^2 \\
=& \ \pi^3 n \left(\sum_{i\in N} \frac{\gamma}{(\gamma - \theta z(\alpha_i))^2} \right) \\
& \left[s^2 \left(\sum_{i\in S} \frac{1}{\gamma - \theta z(\alpha_i)} \right)^2 + \left(\sum_{j\notin S} \frac{1}{\gamma - \theta z(\alpha_j)} \right)^2 + 2s \left(\sum_{i\in S} \frac{1}{\gamma - \theta z(\alpha_i)} \right) \left(\sum_{j\notin S} \frac{1}{\gamma - \theta z(\alpha_j)} \right) \right] \\
&+\pi^2 \left[2ns \left(\sum_{i\in S} \frac{1}{\gamma - \theta z(\alpha_i)} \right) \left(\sum_{i\in N} \frac{\gamma}{(\gamma - \theta z(\alpha_i))^2} \right) + 2n \left(\sum_{j\notin S} \frac{1}{\gamma - \theta z(\alpha_j)} \right) \left(\sum_{i\in N} \frac{\gamma}{(\gamma - \theta z(\alpha_i))^2} \right) \right. \\
&\qquad \left. +s^2 \left(\sum_{i\in S} \frac{1}{\gamma - \theta z(\alpha_i)} \right)^2 + \left(\sum_{j\notin S} \frac{1}{\gamma - \theta z(\alpha_j)} \right)^2 + 2s \left(\sum_{i\in S} \frac{1}{\gamma - \theta z(\alpha_i)} \right) \left(\sum_{j\notin S} \frac{1}{\gamma - \theta z(\alpha_j)} \right) \right] \\
&+\pi \left[n \left(\sum_{i\in N} \frac{\gamma}{(\gamma - \theta z(\alpha_i))^2} \right) + 2s \left(\sum_{i\in S} \frac{1}{\gamma - \theta z(\alpha_i)} \right) + 2 \left(\sum_{j\notin S} \frac{1}{\gamma - \theta z(\alpha_j)} \right) \right] \\
&+1 \\
=&: \ b_3\pi^3 + b_2\pi^2 + b_1\pi + 1
\end{aligned}$$

As an intermediate step, I show $a_1 \geq b_1$ and $a_2 \geq b_2$. Using

$$\frac{1}{\gamma - \theta z(\alpha_i)} = \frac{\gamma - \theta z(\alpha_i)}{(\gamma - \theta z(\alpha_i))^2} \overset{\theta z(\alpha_i)\leq 0}{\geq} \frac{\gamma}{(\gamma - \theta z(\alpha_i))^2}, \tag{A.10}$$

we have

$$
\begin{aligned}
a_1 =& s\left(\sum_{i\in S}\frac{\gamma}{(\gamma-\theta z(\alpha_i))^2}\right)+2n\left(\sum_{i\in N}\frac{1}{\gamma-\theta z(\alpha_i)}\right)\\
=& s\left(\sum_{i\in S}\frac{\gamma}{(\gamma-\theta z(\alpha_i))^2}\right)+2n\left(\left(\sum_{i\in S}\frac{1}{\gamma-\theta z(\alpha_i)}\right)+\left(\sum_{j\notin S}\frac{1}{\gamma-\theta z(\alpha_j)}\right)\right)\\
=& s\left(\sum_{i\in S}\frac{\gamma}{(\gamma-\theta z(\alpha_i))^2}\right)+2(n-s)\left(\sum_{i\in S}\frac{1}{\gamma-\theta z(\alpha_i)}\right)+2s\left(\sum_{i\in S}\frac{1}{\gamma-\theta z(\alpha_i)}\right)\\
&+2(n-1)\left(\sum_{j\notin S}\frac{1}{\gamma-\theta z(\alpha_j)}\right)+2\left(\sum_{j\notin S}\frac{1}{\gamma-\theta z(\alpha_j)}\right)\\
\overset{(A.10)}{\geq}& s\left(\sum_{i\in S}\frac{\gamma}{(\gamma-\theta z(\alpha_i))^2}\right)+2(n-s)\left(\sum_{i\in S}\frac{\gamma}{(\gamma-\theta z(\alpha_i))^2}\right)+2s\left(\sum_{i\in S}\frac{1}{\gamma-\theta z(\alpha_i)}\right)\\
&+2(n-1)\left(\sum_{j\notin S}\frac{\gamma}{(\gamma-\theta z(\alpha_i))^2}\right)+2\left(\sum_{j\notin S}\frac{1}{\gamma-\theta z(\alpha_j)}\right)\\
\overset{n\geq 2}{\geq}& n\left(\sum_{i\in S}\frac{\gamma}{(\gamma-\theta z(\alpha_i))^2}\right)+2s\left(\sum_{i\in S}\frac{1}{\gamma-\theta z(\alpha_i)}\right)\\
&+n\left(\sum_{j\notin S}\frac{\gamma}{(\gamma-\theta z(\alpha_i))^2}\right)+2\left(\sum_{j\notin S}\frac{1}{\gamma-\theta z(\alpha_j)}\right)\\
=& n\left(\sum_{i\in N}\frac{\gamma}{(\gamma-\theta z(\alpha_i))^2}\right)+2s\left(\sum_{i\in S}\frac{1}{\gamma-\theta z(\alpha_i)}\right)+2\left(\sum_{j\notin S}\frac{1}{\gamma-\theta z(\alpha_j)}\right)\\
=& b_1
\end{aligned}
$$

and

$$
\begin{aligned}
a_2 =& 2sn \left(\sum_{i\in N} \frac{1}{\gamma - \theta z(\alpha_i)}\right) \left(\sum_{i\in S} \frac{\gamma}{(\gamma - \theta z(\alpha_i))^2}\right) + n^2 \left(\sum_{i\in N} \frac{1}{\gamma - \theta z(\alpha_i)}\right)^2 \\
=& 2sn \left[\left(\sum_{i\in S} \frac{1}{\gamma - \theta z(\alpha_i)}\right) \left(\sum_{i\in S} \frac{\gamma}{(\gamma - \theta z(\alpha_i))^2}\right) + \left(\sum_{j\notin S} \frac{1}{\gamma - \theta z(\alpha_j)}\right) \left(\sum_{i\in S} \frac{\gamma}{(\gamma - \theta z(\alpha_i))^2}\right)\right] \\
& + n^2 \left[\left(\sum_{i\in S} \frac{1}{\gamma - \theta z(\alpha_i)}\right)^2 + 2\left(\sum_{i\in S} \frac{1}{\gamma - \theta z(\alpha_i)}\right) \left(\sum_{j\notin S} \frac{1}{\gamma - \theta z(\alpha_j)}\right) + \left(\sum_{j\notin S} \frac{1}{\gamma - \theta z(\alpha_j)}\right)^2\right] \\
\geq& 2sn \left(\sum_{i\in S} \frac{1}{\gamma - \theta z(\alpha_i)}\right) \left(\sum_{i\in S} \frac{\gamma}{(\gamma - \theta z(\alpha_i))^2}\right) + 2n \left(\sum_{j\notin S} \frac{1}{\gamma - \theta z(\alpha_j)}\right) \left(\sum_{i\in S} \frac{\gamma}{(\gamma - \theta z(\alpha_i))^2}\right) \\
& + s^2 \left(\sum_{i\in S} \frac{1}{\gamma - \theta z(\alpha_i)}\right)^2 + 2(n^2 - s - sn) \left(\sum_{i\in S} \frac{1}{\gamma - \theta z(\alpha_i)}\right) \left(\sum_{j\notin S} \frac{1}{\gamma - \theta z(\alpha_j)}\right) \\
& + 2s \left(\sum_{i\in S} \frac{1}{\gamma - \theta z(\alpha_i)}\right) \left(\sum_{j\notin S} \frac{1}{\gamma - \theta z(\alpha_j)}\right) + 2sn \left(\sum_{i\in S} \frac{1}{\gamma - \theta z(\alpha_i)}\right) \left(\sum_{j\notin S} \frac{1}{\gamma - \theta z(\alpha_j)}\right) \\
& + (n^2 - 2n - 1) \left(\sum_{j\notin S} \frac{1}{\gamma - \theta z(\alpha_j)}\right)^2 + 2n \left(\sum_{j\notin S} \frac{1}{\gamma - \theta z(\alpha_j)}\right)^2 + \left(\sum_{j\notin S} \frac{1}{\gamma - \theta z(\alpha_j)}\right)^2 \\
\geq& 2sn \left(\sum_{i\in S} \frac{1}{\gamma - \theta z(\alpha_i)}\right) \left(\sum_{i\in S} \frac{\gamma}{(\gamma - \theta z(\alpha_i))^2}\right) + 2sn \left(\sum_{i\in S} \frac{1}{\gamma - \theta z(\alpha_i)}\right) \left(\sum_{j\notin S} \frac{\gamma}{(\gamma - \theta z(\alpha_j))^2}\right) \\
& + 2n \left(\sum_{j\notin S} \frac{1}{\gamma - \theta z(\alpha_j)}\right) \left(\sum_{i\in S} \frac{\gamma}{(\gamma - \theta z(\alpha_i))^2}\right) + 2n \left(\sum_{j\notin S} \frac{1}{\gamma - \theta z(\alpha_j)}\right) \left(\sum_{j\notin S} \frac{\gamma}{(\gamma - \theta z(\alpha_j))^2}\right) \\
& + s^2 \left(\sum_{i\in S} \frac{1}{\gamma - \theta z(\alpha_i)}\right)^2 + 2s \left(\sum_{i\in S} \frac{1}{\gamma - \theta z(\alpha_i)}\right) \left(\sum_{j\notin S} \frac{1}{\gamma - \theta z(\alpha_j)}\right) + \left(\sum_{j\notin S} \frac{1}{\gamma - \theta z(\alpha_j)}\right)^2 \\
=& 2sn \left(\sum_{i\in S} \frac{1}{\gamma - \theta z(\alpha_i)}\right) \left(\sum_{i\in N} \frac{\gamma}{(\gamma - \theta z(\alpha_i))^2}\right) + 2n \left(\sum_{j\notin S} \frac{1}{\gamma - \theta z(\alpha_j)}\right) \left(\sum_{i\in N} \frac{\gamma}{(\gamma - \theta z(\alpha_i))^2}\right) \\
& + s^2 \left(\sum_{i\in S} \frac{1}{\gamma - \theta z(\alpha_i)}\right)^2 + 2s \left(\sum_{i\in S} \frac{1}{\gamma - \theta z(\alpha_i)}\right) \left(\sum_{j\notin S} \frac{1}{\gamma - \theta z(\alpha_j)}\right) + \left(\sum_{j\notin S} \frac{1}{\gamma - \theta z(\alpha_j)}\right)^2
\end{aligned}
$$

$= b_2,$

where

$$
\begin{aligned}
& n^2 - s - sn \geq 0 \quad \text{and} \\
& n^2 - 2n - 1 \geq 0
\end{aligned}
$$

hold for $n \geq 3$ and $s \leq (n-1)$.

To show condition (A.9), I distinguish two cases. If $A \geq B$, then

$$\sum_{S \subset N} \delta_S s \frac{A}{B} \geq \sum_{S \subset N} \delta_S s = \sum_{S \subset N} \delta_S \sum_{i \in N} \mathbb{1}_{i \in S} = \sum_{i \in N} \underbrace{\sum_{S \subset N} \delta_S \mathbb{1}_{i \in S}}_{=1} = n.$$

If $A < B$, then

$$\sum_{S \subset N} \delta_S s \frac{A}{B} = \sum_{S \subset N} \delta_S s \frac{a_3 \pi^3 + a_2 \pi^2 + a_1 \pi + 1}{b_3 \pi^3 + b_2 \pi^2 + b_1 \pi + 1} \geq \sum_{S \subset N} \delta_S s \frac{a_3 \pi^3}{b_3 \pi^3}$$

as

$$a_2 \pi^2 + a_1 \pi + 1 \geq b_2 \pi^2 + b_1 \pi + 1.$$

Further, we have

$$\begin{aligned}
&\sum_{S \subset N} \delta_S s \frac{a_3 \pi^3}{b_3 \pi^3} \\
=&\sum_{S \subset N} \delta_S s \frac{s n^2 \left(\sum_{i \in N} \frac{1}{\gamma - \theta z(\alpha_i)} \right)^2 \left(\sum_{i \in S} \frac{\gamma}{(\gamma - \theta z(\alpha_i))^2} \right)}{n \left(\sum_{i \in N} \frac{\gamma}{(\gamma - \theta z(\alpha_i))^2} \right) \left[s \left(\sum_{i \in S} \frac{1}{\gamma - \theta z(\alpha_i)} \right) + \left(\sum_{j \notin S} \frac{1}{\gamma - \theta z(\alpha_j)} \right) \right]^2} \\
=&\frac{n^2}{n \left(\sum_{i \in N} \frac{\gamma}{(\gamma - \theta z(\alpha_i))^2} \right)} \sum_{S \subset N} \delta_S s^2 \frac{\left(\sum_{i \in N} \frac{1}{\gamma - \theta z(\alpha_i)} \right)^2 \left(\sum_{i \in S} \frac{\gamma}{(\gamma - \theta z(\alpha_i))^2} \right)}{\left[s \left(\sum_{i \in S} \frac{1}{\gamma - \theta z(\alpha_i)} \right) + \left(\sum_{j \notin S} \frac{1}{\gamma - \theta z(\alpha_j)} \right) \right]^2} \\
=&\frac{n}{\left(\sum_{i \in N} \frac{\gamma}{(\gamma - \theta z(\alpha_i))^2} \right)} \sum_{S \subset N} \delta_S \left(\sum_{i \in S} \frac{\gamma}{(\gamma - \theta z(\alpha_i))^2} \right) \frac{\left(s \sum_{i \in N} \frac{1}{\gamma - \theta z(\alpha_i)} \right)^2}{\left[s \left(\sum_{i \in S} \frac{1}{\gamma - \theta z(\alpha_i)} \right) + \left(\sum_{j \notin S} \frac{1}{\gamma - \theta z(\alpha_j)} \right) \right]^2} \\
\geq&\frac{n}{\left(\sum_{i \in N} \frac{\gamma}{(\gamma - \theta z(\alpha_i))^2} \right)} \sum_{S \subset N} \delta_S \left(\sum_{i \in S} \frac{\gamma}{(\gamma - \theta z(\alpha_i))^2} \right) \\
=&\frac{n}{\left(\sum_{i \in N} \frac{\gamma}{(\gamma - \theta z(\alpha_i))^2} \right)} \left(\sum_{i \in N} \frac{\gamma}{(\gamma - \theta z(\alpha_i))^2} \right) \\
=&n,
\end{aligned}$$

showing condition (A.9) and completing the proof. $\square$

A.3 Proof of Proposition 4

Proof. Let $S \subset N, i \in S, j \notin S$. Then j optimizes the certainty equivalent of individual utility.

$$ce_j(U_j(E_j(S), E_N(S))) = P_j^0 - \gamma_j(E_j^0 - E_j(S))^2 - \pi_j E_N(S)^2 + \kappa_j E_N^2 z(\alpha_j)$$
$$0 \stackrel{!}{=} \frac{\partial ce_j(U_j(E_j(S), E_N(S)))}{\partial E_j(S)} = 2\gamma_j(E_j^0 - E_j(S)) - 2\pi_j E_N(S) + 2\kappa_j z(\alpha_j) E_N(S)$$
$$\Rightarrow \quad \gamma_j(E_j^0 - E_j(S)) = (\pi_j - \kappa_j z(\alpha_j)) E_N(S)$$
$$\Rightarrow \quad E_j(S) = E_j^0 - \frac{\pi_j - \kappa_j z(\alpha_j)}{\gamma_j} E_N(S)$$

Player i optimizes the sum of all certainty equivalents of utility of the members of S:

$$\sum_{l \in S} ce_l(U_l(E_l(S), E_N(S))) = \sum_{l \in S} P_l^0 - \gamma_l(E_l^0 - E_l(S))^2 - \pi_l E_N(S)^2 + \kappa_l E_N^2 z(\alpha_l)$$

$$0 \stackrel{!}{=} \frac{\partial \sum_{l \in S} ce_l(U_l(E_l(S), E_N(S)))}{\partial E_i(S)}$$
$$= 2\gamma_i(E_i^0 - E_i(S)) - 2 \underbrace{\sum_{l \in S} \pi_l}_{=:\pi_S} E_N(S) + 2 \underbrace{\sum_{l \in S} \kappa_l z(\alpha_l)}_{=:(\kappa z(\alpha))_S} E_N(S)$$

$$\Rightarrow \quad \gamma_i(E_i^0 - E_i(S)) = (\pi_S - (\kappa z(\alpha))_S) E_N(S)$$
$$\Rightarrow \quad E_i(S) = E_i^0 - \frac{\pi_S - (\kappa z(\alpha))_S}{\gamma_i} E_N(S)$$

The sum of individual emissions of all players gives global emissions $E_N(S)$:

$$E_N(S) = \sum_{l \in N} E_l(S) = \sum_{i \in S} E_i(S) + \sum_{j \notin S} E_j(S)$$
$$= \sum_{i \in S} E_i^0 - \frac{\pi_S - (\kappa z(\alpha))_S}{\gamma_i} E_N(S) + \sum_{j \notin S} E_j^0 - \frac{\pi_j - \kappa_j z(\alpha_j)}{\gamma_j} E_N(S)$$
$$= E_N^0 - \underbrace{\left((\pi_S - (\kappa z(\alpha))_S) \sum_{i \in S} \frac{1}{\gamma_i} + \sum_{j \notin S} \frac{\pi_j - \kappa_j z(\alpha_j)}{\gamma_j} \right)}_{=:\psi(S)} E_N(S)$$

$$\Rightarrow \quad E_N(S) = \frac{E_N^0}{\psi(S) + 1}$$

$\psi(S)$ is the reduction factor of global emissions, showing Proposition 4 (i). The second and third parts of the Proposition follows directly from (i).

□

A.4 Proof of Proposition 6

Proof. As mentioned in Section 2.1, the core of the game is not empty if and only if condition (2.10) is fulfilled.

We have

$$\max_{i\in S} ce_i(V(S)) = \max_{i\in S} \mu(S) + z(\alpha_i)\sigma(S) = \mu(S) + z(\max_{i\in S}\alpha_i)\sigma(S). \tag{A.11}$$

For simplicity, define

$$z_{max}(S) := z(\max_{i\in S}\alpha_i), (z\kappa)_S := \sum_{i\in S} z_i\kappa_i, \left(\frac{1}{\gamma}\right)_S := \sum_{i\in S}\frac{1}{\gamma_i}.$$

Then condition (2.10) is equivalent to

$$P_N^0 - \left(\pi_N + (\pi_N - (z\kappa)_N)^2 \left(\frac{1}{\gamma}\right)_N\right) E_N(N)^2 + z_{max}(N)\sqrt{\sum_{i\in N}\kappa_i^2 E_N(N)^2}$$

$$\geq \sum_{S\subset N}\delta_S\left[P_S^0 - \left(\pi_S + (\pi_S - (z\kappa)_S)^2\left(\frac{1}{\gamma}\right)_S\right) E_N(S)^2 + z_{max}(S)\sqrt{\sum_{i\in S}\kappa_i^2 E_N(S)^2}\right]$$

$$\Leftrightarrow \quad \left[\pi_N - z_{max}(N)\sqrt{\sum_{i\in N}\kappa_i^2} + (\pi_N - (z\kappa)_N)^2\left(\frac{1}{\gamma}\right)_N\right] E_N(N)^2$$

$$\leq \sum_{S\subset N}\delta_S\left[\pi_S - z_{max}(S)\sqrt{\sum_{i\in S}\kappa_i^2} + (\pi_S - (z\kappa)_S)^2\left(\frac{1}{\gamma}\right)_S\right] E_N(S)^2$$

$$\Leftrightarrow \quad \pi_N - z_{max}(N)\sqrt{\sum_{i\in N}\kappa_i^2} + (\pi_N - (z\kappa)_N)^2\left(\frac{1}{\gamma}\right)_N$$

$$\leq \sum_{S\subset N}\delta_S\left[\pi_S - z_{max}(S)\sqrt{\sum_{i\in S}\kappa_i^2} + (\pi_S - (z\kappa)_S)^2\left(\frac{1}{\gamma}\right)_S\right]\left(\frac{\psi(N)+1}{\psi(S)+1}\right)^2$$

$$\Leftrightarrow \quad \pi_N - z_{max}(N)\sqrt{\sum_{i\in N}\kappa_i^2} + (\pi_N - (z\kappa)_N)^2\left(\frac{1}{\gamma}\right)_N$$

$$\leq \sum_{S\subset N}\delta_S\left[\pi_S - z_{max}(S)\sqrt{\sum_{i\in S}\kappa_i^2} + (\pi_S - (z\kappa)_S)^2\left(\frac{1}{\gamma}\right)_S\right] \tag{A.12}$$

$$\left(\frac{(\pi_N - (z\kappa)_N)\left(\frac{1}{\gamma}\right)_N + 1}{(\pi_S - (z\kappa)_S)\left(\frac{1}{\gamma}\right)_S + \sum_{j\notin S}\frac{\pi_j - z_j\kappa_j}{\gamma_j} + 1}\right)^2$$

I first show condition (A.12) for the special case of the *All Singletons* map, i.e.

$$\delta_S^{\text{Singl}} = \begin{cases} 1, & S = \{i\}, i\in N \\ 0, & \text{otherwise.}\end{cases} \tag{A.13}$$

Then the right hand side of (A.12) is equal to

$$\sum_{i\in N}\frac{\left(\pi_i - z_i\kappa_i + \frac{(\pi_i-z_i\kappa_i)^2}{\gamma_i}\right)\left((\pi_N-(z\kappa)_N)\left(\frac{1}{\gamma}\right)_N+1\right)^2}{\left(\sum_{l\in N}\frac{\pi_l-z_l\kappa_l}{\gamma_l}+1\right)^2}$$
$$=\left((\pi_N-(z\kappa)_N)\left(\frac{1}{\gamma}\right)_N+1\right)\sum_{i\in N}\frac{(\pi_i-z_i\kappa_i)\left(1+\frac{\pi_i-z_i\kappa_i}{\gamma_i}\right)\left((\pi_N-(z\kappa)_N)\left(\frac{1}{\gamma}\right)_N+1\right)}{\left(\sum_{l\in N}\frac{\pi_l-z_l\kappa_l}{\gamma_l}+1\right)^2}$$

I now show

$$\sum_{i\in N}\frac{(\pi_i-z_i\kappa_i)\left(1+\frac{\pi_i-z_i\kappa_i}{\gamma_i}\right)\left((\pi_N-(z\kappa)_N)\left(\frac{1}{\gamma}\right)_N+1\right)}{\left(\sum_{l\in N}\frac{\pi_l-z_l\kappa_l}{\gamma_l}+1\right)^2}\geq \pi_N-(z\kappa)_N, \tag{A.14}$$

which implies (A.12), as

$$\begin{aligned}&\left((\pi_N-(z\kappa)_N)\left(\frac{1}{\gamma}\right)_N+1\right)(\pi_N-(z\kappa)_N)\\ =&(\pi_N-(z\kappa)_N)^2\left(\frac{1}{\gamma}\right)_N+\pi_N-(z\kappa)_N\\ \geq&(\pi_N-(z\kappa)_N)^2\left(\frac{1}{\gamma}\right)_N+\pi_N-z_{max}(N)\sqrt{\sum_{i\in N}\kappa_i^2}\end{aligned}$$

For simplicity, define $\tau_i := \pi_i - z_i\kappa_i$. Then (A.14) is equivalent to

$$\sum_{i\in N}\frac{\tau_i\left(1+\frac{\tau_i}{\gamma_i}\right)\left(\tau_N\left(\frac{1}{\gamma}\right)_N+1\right)}{\left(\sum_{l\in N}\frac{\tau_l}{\gamma_l}+1\right)^2}\geq\tau_N$$
$$\Leftrightarrow\sum_{i\in N}\tau_i\left(1+\frac{\tau_i}{\gamma_i}\right)\left(\tau_N\left(\frac{1}{\gamma}\right)_N+1\right)\geq\tau_N\left(\sum_{l\in N}\frac{\tau_l}{\gamma_l}+1\right)^2$$

We have

$$
\begin{aligned}
&\sum_{i\in N}\tau_i\left(1+\frac{\tau_i}{\gamma_i}\right)\left(\tau_N\left(\frac{1}{\gamma}\right)_N+1\right)\\
=&\sum_{i\in N}\tau_i\left(1+\frac{\tau_i}{\gamma_i}\right)\left(\sum_{j\in N}\tau_j\sum_{k\in N}\frac{1}{\gamma_k}+1\right)\\
=&\sum_{i,j,k\in N}\frac{\tau_i^2\tau_j}{\gamma_i\gamma_k}+\sum_{i,j,k\in N}\frac{\tau_i\tau_j}{\gamma_k}+\sum_{i\in N}\frac{\tau_i^2}{\gamma_i}+\sum_{i\in N}\tau_i\\
=&\sum_{i,j\in N}\frac{\tau_i^2\tau_j}{\gamma_i^2}+\sum_{\substack{i,j,k\in N\\k\neq i}}\frac{\tau_i^2\tau_j}{\gamma_i\gamma_k}+\sum_{i,j\in N}\frac{\tau_i\tau_j}{\gamma_i}+\sum_{i,j\in N}\frac{\tau_i\tau_j}{\gamma_j}+\sum_{\substack{i,j,k\in N\\k\neq i\\k\neq j}}\frac{\tau_i\tau_j}{\gamma_k}+\sum_{i\in N}\frac{\tau_i^2}{\gamma_i}+\sum_{i\in N}\tau_i\\
\geq&\sum_{i,j\in N}\frac{\tau_i^2\tau_j}{\gamma_i^2}+\sum_{\substack{i,j,k\in N\\k\neq i}}\frac{\tau_i\tau_k\tau_j}{\gamma_i\gamma_k}+2\sum_{i,j\in N}\frac{\tau_i\tau_j}{\gamma_i}+\sum_{i\in N}\tau_i\\
=&\sum_{i\in N}\tau_i\left(\sum_{j\in N}\frac{\tau_j^2}{\gamma_j^2}+\sum_{\substack{j\in N\\k\neq j}}\frac{\tau_j\tau_k}{\gamma_j\gamma_k}+2\sum_{j\in N}\frac{\tau_j}{\gamma_j}+1\right)\\
=&\sum_{i\in N}\tau_i\left(\left(\sum_{j\in N}\frac{\tau_j}{\gamma_j}\right)^2+2\sum_{j\in N}\frac{\tau_j}{\gamma_j}+1\right)\\
=&\tau_N\left(\sum_{l\in N}\frac{\tau_l}{\gamma_l}+1\right)^2,
\end{aligned}
$$

showing (A.14).

For the general case, define

$$
g(S):=\frac{\pi_S-z_{max}(S)\sqrt{\sum_{l\in S}\kappa_l^2}+(\pi_S-(z\kappa)_S)^2\left(\frac{1}{\gamma}\right)_S}{\left[(\pi_S-(z\kappa)_S)\left(\frac{1}{\gamma}\right)_S+\sum_{l\notin S}\frac{\pi_l-z_l\kappa_l}{\gamma_l}+1\right]^2}. \tag{A.15}
$$

From the singleton case, we have

$$
\begin{aligned}
&\left((\pi_N-(z\kappa)_N)\left(\frac{1}{\gamma}\right)_N+1\right)^2\sum_{S\subset N}\delta_S^{\text{Singl}}g(S)\\
\geq&(\pi_N-(z\kappa)_N)^2\left(\frac{1}{\gamma}\right)_N+\pi_N-z_{max}(N)\sqrt{\sum_{i\in N}\kappa_i^2}.
\end{aligned} \tag{A.16}
$$

Let

$$\hat{\delta}_S = \begin{cases} \epsilon, & S = \{i,j\} \\ 1-\epsilon, & S = \{i\}, S = \{j\} \\ \delta_S^{\text{Singl}}, & \text{otherwise} \end{cases} \tag{A.17}$$

for some $\epsilon > 0$. $\hat{\delta}$ thus is a balanced map that shifts some weight from the singleton coalitions of the players i and j to the joint coalition of these two players. I now show that $\hat{\delta}$ also satisfies condition (A.12). Two cases need to be distinguished:

$$\text{Case 1:} \qquad g(\{i,j\}) \geq g(\{i\}) + g(\{j\}) \tag{A.18}$$

Then

$$\begin{aligned} &\left((\pi_N - (z\kappa)_N)\left(\frac{1}{\gamma}\right)_N + 1\right)^2 \sum_{S\subset N} \hat{\delta}_S g(S) \\ \overset{(A.18)}{\geq} &\left((\pi_N - (z\kappa)_N)\left(\frac{1}{\gamma}\right)_N + 1\right)^2 \sum_{S\subset N} \delta_S^{\text{Singl}} g(S) \\ \overset{(A.16)}{\geq} &(\pi_N - (z\kappa)_N)^2 \left(\frac{1}{\gamma}\right)_N + \pi_N - z_{max}(N)\sqrt{\sum_{i\in N}\kappa_i^2}, \end{aligned}$$

showing (A.12).

$$\text{Case 2:} \qquad g(\{i,j\}) < g(\{i\}) + g(\{j\}) \tag{A.19}$$

Define a new game, denoted by $\sim$, in which players i and j are replaced by a single player, called T, i.e.

$$\tilde{N} = N \backslash \{i,j\} \cup \{T\}. \tag{A.20a}$$

Let

$$\tilde{z}_T := z_{max}(\{i,j\}), \tag{A.20b}$$

$$\tilde{\kappa}_T := \sqrt{\kappa_i^2 + \kappa_j^2}, \tag{A.20c}$$

$$\tilde{E}_T^0 := E_i^0 + E_j^0, \tag{A.20d}$$

$$\tilde{P}_T^0 := P_i^0 + P_j^0, \tag{A.20e}$$

while the parameters for other players stay as in the original game.

Choose $\tilde{\pi}_T$ and $\tilde{\gamma}_T$ as solutions to the system of inequalities

$$\left((\pi_N - (z\kappa)_N) \left(\frac{1}{\gamma}\right)_N + 1 \right)^2 \left[g(\{i,j\}) + \sum_{\substack{l \in N \\ l \neq i,j}} g(\{l\}) \right]$$

$$\geq \left((\tilde{\pi}_{\tilde{N}} - (\tilde{z}\tilde{\kappa})_{\tilde{N}}) \left(\frac{1}{\tilde{\gamma}}\right)_{\tilde{N}} + 1 \right)^2 \left[\tilde{g}(T) + \sum_{\substack{l \in N \\ l \neq i,j}} \tilde{g}(\{l\}) \right], \tag{A.21a}$$

$$(\tilde{\pi}_{\tilde{N}} - (\tilde{z}\tilde{\kappa})_{\tilde{N}}))^2 \left(\frac{1}{\tilde{\gamma}}\right)_{\tilde{N}} + \tilde{\pi}_{\tilde{N}} \geq (\pi_N - (z\kappa)_N)^2 \left(\frac{1}{\gamma}\right)_N + \pi_N. \tag{A.21b}$$

For example, in Example 2, one could choose $\tilde{\pi}_T$ and $\tilde{\gamma}_T$ as in Table 3. A general solution for $\tilde{\pi}_T$ and $\tilde{\gamma}_T$ is not spelled out in this proof, due to length and complexity.

i	j	$\tilde{\pi}_T$	$\tilde{\gamma}_T$
1	2	2.00E-3	5.56E-2
1	3	2.00E-1	1.00E-1
2	3	1.50E-1	1.00E3

Table 3: Values of $\tilde{\pi}_T$ and $\tilde{\gamma}_T$ that satisfy equations (A.21) in Example 2.

Thus

$$
\begin{aligned}
&\left((\pi_N - (z\kappa)_N)\left(\frac{1}{\gamma}\right)_N + 1\right)^2 \sum_{S\subset N} \hat{\delta}_S g(S) \\
&= \left((\pi_N - (z\kappa)_N)\left(\frac{1}{\gamma}\right)_N + 1\right)^2 \left[(1-\epsilon)g(\{i,j\}) + \epsilon(g(\{i\}) + g(\{j\})) + \sum_{\substack{l\in N\\ l\neq i,j}} g(\{l\})\right] \\
&\overset{(A.19)}{\geq} \left((\pi_N - (z\kappa)_N)\left(\frac{1}{\gamma}\right)_N + 1\right)^2 \left[g(\{i,j\}) + \sum_{\substack{l\in N\\ l\neq i,j}} g(\{l\})\right] \\
&\overset{(A.21a)}{\geq} \left((\tilde{\pi}_{\tilde{N}} - (\tilde{z}\tilde{\kappa})_{\tilde{N}})\left(\frac{1}{\tilde{\gamma}}\right)_{\tilde{N}} + 1\right)^2 \left[\tilde{g}(T) + \sum_{\substack{l\in N\\ l\neq i,j}} \tilde{g}(\{l\})\right] \\
&= \left((\tilde{\pi}_{\tilde{N}} - (\tilde{z}\tilde{\kappa})_{\tilde{N}})\left(\frac{1}{\tilde{\gamma}}\right)_{\tilde{N}} + 1\right)^2 \sum_{S\subset \tilde{N}} \tilde{\delta}_S^{\text{Singl}} \tilde{g}(S) \\
&\overset{(A.16)}{\geq} (\tilde{\pi}_{\tilde{N}} - (\tilde{z}\tilde{\kappa})_{\tilde{N}}))^2 \left(\frac{1}{\tilde{\gamma}}\right)_{\tilde{N}} + \tilde{\pi}_{\tilde{N}} - z_{max}(N)\sqrt{\sum_{i\in N}\kappa_i^2} \\
&\overset{(A.21b)}{\geq} (\pi_N - (z\kappa)_N)^2 \left(\frac{1}{\gamma}\right)_N + \pi_N - z_{max}(N)\sqrt{\sum_{i\in N}\kappa_i^2},
\end{aligned}
$$

showing (A.12). As the iterative application of transformation (A.17) can produce any balanced map δ, this completes the proof. □

B Additional tables

Coalition S	V(S)	
	μ	σ^2
{1}	8.7498	0.0050
{2}	8.5937	0.0514
{3}	7.6406	1.3917
{4}	7.6406	1.3917
{1,2}	17.4751	0.4565
{1,3}	15.8882	6.9740
{1,4}	15.8882	6.9740
{2,3}	15.8706	6.1752
{2,4}	15.8706	6.1752
{3,4}	15.0455	7.8550
{1,2,3}	25.4051	8.7419
{1,2,4}	25.4051	8.7419
{1,3,4}	24.8395	9.3169
{2,3,4}	25.0362	7.8691
{1,2,3,4}	35.3460	7.0608

Table 4: Cooperative game with stochastic payoffs for parameters (3.8). Payoffs are normally distributed with $\mathcal{N}(\mu, \sigma^2)$.

Coalition S	V(S)	
	μ	σ^2
{1}	3.5E+03	3E+05
{2}	4.84E+04	3.11E+07
{3}	4.99E+04	7.783E+08
{1,2}	5.03E+04	1.46E+07
{1,3}	5.38E+04	5.244E+08
{2,3}	9.77E+04	5.340E+08
{1,2,3}	1.009E+05	1.962E+08

Table 5: Cooperative game with stochastic payoffs for parameters (4.6). Payoffs are normally distributed with $\mathcal{N}(\mu, \sigma^2)$.

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