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Y. Efendiev, O. Iliev, C. Kronsbein

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Berichte des Fraunhofer ITWM, Nr. 217 (2012)

ISSN 1434-9973

Bericht 217 (2012)

Die Veröffentlichungen in der Berichtsreihe des Fraunhofer ITWM können bezogen werden über:

Fraunhofer-Institut für Techno- und
Wirtschaftsmathematik ITWM
Fraunhofer-Platz 1

67663 Kaiserslautern
Germany

Telefon: +49(0)631/3160 0-4674
Telefax: +49(0)631/3160 0-5674
E-Mail: presse@itwm.fraunhofer.de
Internet: www.itwm.fraunhofer.de

Vorwort

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Darüber hinaus bietet die Reihe ein Forum für die Berichterstattung über die zahlreichen Kooperationsprojekte des Instituts mit Partnern aus Industrie und Wirtschaft.

Berichterstattung heißt hier Dokumentation des Transfers aktueller Ergebnisse aus mathematischer Forschungs- und Entwicklungsarbeit in industrielle Anwendungen und Softwareprodukte – und umgekehrt, denn Probleme der Praxis generieren neue interessante mathematische Fragestellungen.

Prof. Dr. Dieter Prätzel-Wolters
Institutsleiter

Kaiserslautern, im Juni 2001

Multi-level Monte Carlo methods using ensemble level mixed MsFEM for two-phase flow and transport simulations

Y. Efendiev* O. Iliev† C. Kronsbein‡

May 30, 2012

Abstract

In this paper, we propose multi-level Monte Carlo (MLMC) methods that use ensemble level mixed multiscale methods in the simulations of multi-phase flow and transport. The main idea of ensemble level multiscale methods is to construct local multiscale basis functions that can be used for any member of the ensemble. We consider two types of ensemble level mixed multiscale finite element methods, (1) the no-local-solve-online ensemble level method (NLSO) and (2) the local-solve-online ensemble level method (LSO). Both mixed multiscale methods use a number of snapshots of the permeability media to generate a multiscale basis. As a result, in the offline stage, we construct multiple basis functions for each coarse region where basis functions correspond to different realizations. In the no-local-solve-online ensemble level method one uses the whole set of pre-computed basis functions to approximate the solution for an arbitrary realization. In the local-solve-online ensemble level method one uses the pre-computed functions to construct a multiscale basis for a particular realization. With this basis the solution corresponding to this particular realization is approximated in LSO mixed MsFEM. In both approaches the accuracy of the method is related to the number of snapshots computed based on different realizations that one uses to pre-compute a multiscale basis. We note that LSO approaches share similarities with reduced basis methods [11, 21, 22].

In multi-level Monte Carlo methods ([14, 13]), more accurate (and expensive) forward simulations are run with fewer samples while less accurate (and inexpensive) forward simulations are run with a larger number of samples. Selecting the number of expensive and inexpensive simulations

*Department of Mathematics, Texas A&M University, College Station, TX 77843-3368. email: efendiev@math.tamu.edu. Corresponding author

†Fraunhofer ITWM & University of Kaiserslautern, 67663 Kaiserslautern, Germany. email: iliev@itwm.fraunhofer.de

‡Fraunhofer ITWM & University of Kaiserslautern, 67663 Kaiserslautern, Germany. email: kronsbein@itwm.fraunhofer.de

carefully, one can show that MLMC methods can provide better accuracy at the same cost as MC methods.

In our simulations, our goal is twofold. First, we would like to compare NLSO and LSO mixed MsFEMs. In particular, we show that NLSO mixed MsFEM is more accurate compared to LSO mixed MsFEM. Further, we use both approaches in the context of MLMC to speed-up MC calculations. We present basic aspects of the algorithm and numerical results for coupled flow and transport in heterogeneous porous media.

1 Introduction

Multi-phase flow and transport simulations in heterogeneous subsurface formations are challenging due to a rich hierarchy of spatial scales and uncertainties. Typical approaches include upscaling or multiscale methods where the solution is approximated on a coarse grid [8, 12]. In these simulations, effective media properties or multiscale basis functions are constructed for each coarse-grid block and they are used to inexpensively solve the problem on a coarse grid. Coarse-grid simulations help to reduce the computational cost; however, they can still be very expensive for stochastic problems due to the number of Monte Carlo (MC) simulations that are needed to make accurate predictions. Indeed, each MC simulation requires a coarse-grid parameter calculations that can be expensive, especially when one needs to generate relative permeabilities or other nonlinear constitutive relations. For this reason, various techniques are proposed to reduce the number of simulations in Monte Carlo methods. These approaches include ensemble level upscaling [7] or multiscale methods [3] that use fewer samples to compute effective properties for the whole ensemble. It is expensive to apply multiscale methods directly because basis functions need to be computed for each realization.

In this paper, we discuss two ensemble level mixed multiscale finite element methods (1) the no-local-solve-online (NLSO) and (2) the local-solve-online ensemble level method (LSO). The main idea of the ensemble level methods is to pre-compute multiscale basis functions using a number of permeability realizations. In the offline stage, in each coarse-grid block, we compute multiscale basis functions by selecting some realizations from the ensemble. Using these multiscale basis functions, our goal is to approximate the solution for an arbitrary realization. In the LSO method we use the pre-computed functions to build a multiscale basis in each coarse block for each arbitrary realization at the online stage. This approach entails of solving a local problem at the online stage and, therefore, we call it the local-solve-online mixed MsFEM. For the local problems we have to impose boundary conditions. The main idea of NLSO is to use all pre-computed basis functions to construct a coarse grid approximation. This approach does not compute a new multiscale basis function at the online stage and instead solves a larger coarse-grid system using all pre-computed multiscale basis functions.

The advantage of NLSO approaches is more evident if limited global information [4] is used to construct multiscale basis functions. As it was shown

(e.g., [4]), one can use single-phase flow information to construct multiscale basis functions and this can provide several-fold improvement in the accuracy in two-phase flow simulations. In online stage one needs boundary conditions for basis functions in the case of LSO. On the other hand, NLSO does not require any local solves and thus we can take an advantage of accurate multiscale basis functions computed for selected realizations. For LSO, even if multiscale basis functions at the offline stage are computed with limited global information, in the online stage, we impose local boundary conditions which affect the accuracy of the solution as we show in the paper. Note that LSO approaches are less expensive and thus we consider them.

In both methods the accuracy of the approximation of the solution for an arbitrary realization depends on the dimension of the pre-computed basis space. The more multiscale basis functions we pre-compute the more accurate is the approximation. In NLSO, one does not need to generate a coarse space for each new selected realization and simply projects the global solution onto the ensemble level multiscale space. For both method, NLSO and LSO, respectively, we consider boundary conditions for the pre-computation which use local or global information. We compare the NLSO and the LSO methods in terms of accuracy.

We apply the ensemble level methods in the framework of multi-level Monte Carlo methods (MLMC) to reduce the computational costs in comparison with a Monte Carlo method. Multi-level Monte Carlo, introduced by Heinrich ([15]) and later applied to stochastic ODEs by Giles ([14, 13]), and PDEs with stochastic coefficients by Schwab et al.([6]) and Cliffe et al. ([10]). The main idea of multi-level Monte Carlo (MLMC) is to use a number of samples at different levels to compute the expected values of quantities of interest. In these techniques, more realizations are used at the coarsest level with inexpensive forward computations. At the same time, fewer samples are used at the finest, most expensive to compute level. Combining the results of these computations by choosing the number of realizations at each level carefully, one can speed-up the computations. In this paper, we take the dimension of the multiscale space to be variable in MLMC.

In this paper, we take different numbers of samples to generate the multiscale space. We show that the NLSO mixed MsFEM using global information gives more accurate approximation than the LSO mixed MsFEM. More forward coarse-grid simulations are run using the smaller dimensional multiscale spaces while less simulations are run using higher dimensional multiscale spaces. We show that by combining these simulation results in a MLMC framework one can achieve better accuracy for the same cost as MC. For this combination, one needs to know the convergence of ensemble level methods with respect to the coarse space dimension, respectively. One can estimate this rate based on a small number of apriori computations as discussed in the paper.

We present numerical results where single-phase flow and transport as well as two-phase flow and transport are tested. Permeability fields in our simulations are described by two-point correlation functions and parameterized using Karhunen-Loève expansion. In the simulations, we compare saturation profiles

at certain time instants and run MC computations on the finest coarse model such that the cost of MC and MLMC computations are the same. Our numerical results show that one can achieve higher accuracy when MLMC is used. We also compare the accuracy of NLSO and LSO mixed MsFEM. Though LSO mixed MsFEM is computationally more efficient, it is less accurate compared to NLSO mixed MsFEM.

The paper is organized as follows. In the next section, we present the model problem and the mixed multiscale finite element method. In Section 3 we introduce the ensemble level mixed MsFEMs, no-local-solve-online (NLSO) and local-solve-online ensemble level mixed MsFEM (LSO). Section 4 is devoted to MLMC using multiscale methods. We present numerical results in Section 5.

2 Preliminaries

In this paper, we will consider two-phase flow and transport in a porous media under the assumption that the displacement is dominated by viscous effects; i.e., we neglect the effects of gravity, compressibility, and capillary pressure. The two phases will be referred to as water and oil, designated by subscripts w and o, respectively. The governing equations describing this system are given by coupled pressure and saturation equations (saturation for the water phase)

$$-\operatorname{div}(\lambda(S)k\nabla p) = q_w + q_o, \tag{2.1}$$

$$\phi\frac{\partial S}{\partial t} + \nabla(vf(S)) = q_w, \tag{2.2}$$

where λ is the total mobility, $f(S)$ is the flux function, and v is the total velocity, which are

$$\lambda(S) = \frac{k_{rw}(S)}{\mu_w} + \frac{k_{ro}(S)}{\mu_o}, \quad f(S) = \frac{k_{rw}(S)}{\mu_w\lambda(S)}, \quad v = -\lambda(S)k\cdot\nabla p, \tag{2.3}$$

where S is the water saturation (volume fraction), p is pressure, q_w and q_o are volumetric source terms for water and oil, and ϕ is the porosity. Here, $k_{rw}(S)$ and $k_{ro}(S)$ are relative permeabilities of water and oil phases that appear in Darcy's law for each phase $v_j = -\frac{k_{rj}(S)}{\mu_j}k\cdot\nabla p$, where v_j is the phase velocity, k is the permeability tensor and μ_j is the phase viscosity. We use a single set of relative permeability curves.

2.1 Mixed multiscale finite element methods

In this section, we discuss mixed multiscale finite element methods (MsFEM) following [2, 12]. We write the two-phase flow equation as

$$\begin{aligned} (\lambda k)^{-1}v + \nabla p &= 0 \quad \text{in} \quad \Omega \\ \operatorname{div}(v) &= q \quad \text{in} \quad \Omega \\ \lambda k\nabla p\cdot n &= g \quad \text{on} \quad \partial\Omega. \end{aligned} \tag{2.4}$$

We assume Neumann boundary conditions.

Let $V_h \subset H(\mathrm{div}, \Omega)$ and $Q_h \subset L^2(\Omega)/R$ be finite dimensional spaces and $V_h^0 = V_h \cap H_0(\mathrm{div}, \Omega)$, where $H_0(\mathrm{div}, \Omega)$ is $H(\mathrm{div}, \Omega)$ with homogeneous boundary conditions. The numerical approximation of (2.4) on the fine grid is to find $\{v_h, p_h\} \in V_h \times Q_h$ such that $v_h \cdot n = g_h$ on $\partial\Omega$ and

$$\begin{array}{rcll} ((\lambda k)^{-1} v_h, u_h) - (\mathrm{div} u_h, p_h) & = & 0 & \forall u_h \in V_h^0 \\ (\mathrm{div} v_h, b_h) & = & (q, b_h) & \forall b_h \in Q_h, \end{array} \tag{2.5}$$

with the usual L^2 inner product, $(\cdot, \cdot)$.

In a mixed MsFEM one attempts to design the approximation space for velocity in such a way that it contains the small-scale features of the solution. In a mixed MsFEM, the velocity field is approximated using multiscale basis functions, while piecewise constant functions are used to approximate the pressure field. In particular, these multiscale basis functions for the velocity field are constructed for each edge (face) of every block. Let $\bigcup K_i = \Omega$ be a partitioning of the domain in polyhedral elements. We denote with $\mathcal{I}$ the multi index set of pairs of indices of two neighboring blocks, i.e if $K_i \cap K_{i'} \neq \emptyset$, $\iota = \{ii'\} \in \mathcal{I}$. We denote the interface of two neighboring blocks K_i and $K_{i'}$ with Γ_ι , i.e Γ_ι is a common edge or a face of the blocks K_i and $K_{i'}$.
With a given permeability field k, we compute the corresponding multiscale basis function for Γ_ι in the following way.

First we compute a function $w_{\iota,k}$ by solving the following equation:

$$\begin{array}{rcl} (\text{-}\mathrm{div}(k(x)\nabla w_{\iota,k}))\,|_{K_i} = & \left\{ \begin{array}{ll} \frac{1}{|K_i|} & \text{if } \int_{K_i} q = 0 \\ \frac{q}{\int_{K_i} q} & \text{else,} \end{array} \right. \\ (-\mathrm{div}(k(x)\nabla w_{\iota,k}))\,|_{K_{i'}} = & \left\{ \begin{array}{ll} \frac{-1}{|K_{i'}|} & \text{if } \int_{K_{i'}} q = 0 \\ \frac{-q}{\int_{K_{i'}} q} & \text{else,} \end{array} \right. \\ -k(x)\nabla w_{\iota,k} \cdot n_{ii'} = & \left\{ \begin{array}{ll} g_\iota & \text{on } \Gamma_\iota \\ 0 & \text{else,} \end{array} \right. \end{array} \tag{2.6}$$

where the choice of g_ι will be discussed later and $n_{ii'}$ is the normal pointing from K_i to $K_{i'}$ (see Figure 1).

To each auxiliary function $w_{\iota,k}$, we associate the multiscale basis function $\Psi_{\iota,k} = -k(x)\nabla w_{\iota,k}$. When basis functions have been computed for each edge, we define the finite dimensional space for approximation of the velocity

$$\begin{array}{rcl} V_h(k) & = & \bigoplus_\iota \{\Psi_{\iota,k}\}, \\ V_h^0(k) & = & V_h(k) \cap H_0(\mathrm{div}, \Omega). \end{array}$$

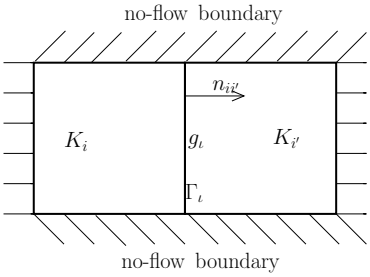


Figure 1: Local problem to solve for multiscale basis.

Note that for each permeability field k, we associate a velocity approximation space $V_h(k)$ that is spanned by the set of basis functions consisting of one multiscale basis function for each edge of every grid block in the coarse grid. In Section 3 we introduce to different approximation spaces for the velocity.

The choice of the boundary conditions g_ι in (2.6) can affect the accuracy of MsFEM. In the following subsections we introduce two different kinds of boundary conditions, local and global boundary conditions where for global boundary conditions, we employ a single-phase flow velocity field.

2.2 Mixed MsFEM using local information

In [9], piecewise constant coarse-scale fluxes on the boundary of the coarse elements are used, i.e.,

$$g_\iota = \frac{1}{|\Gamma_\iota|}.$$

In this case, the boundary conditions do not contain any fine-scale information that is in the velocity field of the reference solution. In general, we also consider as a local approach any other boundary conditions that involve local information, e.g., permeabilities within the local domain.

2.3 Mixed MsFEM using limited global information

For accurate approximations of the fluxes, the velocity should contain fine-scale features similar to the solution. Approaches such as oversampling or limited global information [16, 12] are introduced for mixed finite element methods. The main idea of these techniques is to use larger regions for local problem computations, i.e., the local problems are solved in larger regions and then the basis functions are computed using only the interior information. In the following we use

$$g_\iota = \frac{v \cdot n_{ii'}}{\int_{\Gamma_\iota} v \cdot n_{ii'} ds},$$

with the 'global' velocity v which solves the single-phase flow equation (2.4).

Here, we note that global approaches are known to be more accurate for a given realization of the permeability field ([4]). However, they are computationally more expensive, since the boundary condition depends on the 'global' velocity v which requires to solve a global fine-scale problem.

3 Ensemble level methods for mixed MsFEM

The main idea of ensemble level multiscale methods is to construct local basis functions that can be used for any member of the ensemble. These calculations involve selecting some ensemble members and constructing multiscale basis functions for them. The computations are divided in offline and online computations. In this section we introduce two ensemble level methods for mixed MsFEM, the no-local-solve-online ensemble level method (NLSO) and the local-solve-online ensemble level method (NLSO). In both methods the offline part is to construct sets of basis functions based on a few (N_l) realizations of the solution. We choose these realizations randomly or use proper orthogonal decomposition (POD) to identify best local basis functions in the norm defined by POD for the velocity field. In general we can also choose offline realizations following the techniques used in reduced basis methods (cf.[21, 22]). For both methods we can use either boundary conditions using global single-phase flow information or local information.

The main idea of NLSO is to use all pre-computed basis functions to construct a coarse-grid approximation. This is done by projecting the solution onto the space of pre-computed basis functions. On the other hand, LSO computes a few (one in our case) multiscale basis functions per edge by projecting the local problem onto the space of pre-computed basis functions. The local problem typically requires some boundary conditions. For a given realization, we can only use local boundary conditions because the use of limited global information will involve solving a global single-phase flow equation. For this reason, as we will show that LSO is less accurate compared to NLSO when global information is used.

3.1 No-local-solve-online ensemble level method (NLSO)

The no-local-solve-online mixed MsFEM method was introduced in [3]. The main idea is to pre-compute multiscale basis functions on a coarse-grid block for a few randomly chosen realizations. These basis functions are further used to solve the global flow equation (2.4). In particular, at each interior edge Γ_ι and for a few coefficients $k_j(x) = k(x, \omega_j)$, $1 \leq j \leq N_l$, we define the multiscale basis functions $\Psi_{\iota,k_j}(x) = -k_j(x)\nabla w_{\iota,k_j}$. Whereas w_{ι,k_j} denotes the solution of (2.6) with the coefficient $k_j(x)$.

These basis functions are used to solve the equation on a coarse grid for an arbitrary realization of the permeability without re-computing new basis functions. Following [3], we employ an approximation space for the velocity

that is defined for multiple k_j, i.e., we use

$$V_h^{NLSO} = \bigoplus_{j=1}^{N_l} V_h(k_j).$$

The coarse space of all the pre-computed basis functions is used to solve the equation (2.4) on a coarse grid. This velocity we use to solve the transport equation (2.2) with an implicit scheme to determine the saturation.

Therefore we solve the equations on a $N_l \times |\mathcal{I}|$ coarse space (since we have N_l basis functions on each edge instead of one).

Since the basis functions do not change during the online stage, we can pre-compute the integrals such as $\int \Psi_{\iota,k_j}\Psi_{\iota',k'_j}$.

To identify the best local basis we use proper orthogonal decomposition. Therefore we compute the $\tilde{N}$ eigenvalues with the largest absolute value and the corresponding eigenvectors $\tilde{V}_i = (\tilde{V}_i^1, \cdots, \tilde{V}_i^{N_l})$, $1 \leq i \leq \tilde{N}$, of the matrix $B_\iota^T B_\iota$. Here B_ι denotes the matrix of pre-computed multiscale basis functions for an edge ι, i.e., of $B_\iota = \left(\Psi_{\iota,k_j}\right)_{1\leq j\leq N_l}$. We denote with V the matrix of the scaled eigenvectors, i.e., for each column of the matrix it holds: $V_i = \frac{1}{\sum_{j=1}^{N_l} \tilde{V}_i^j}\tilde{V}_i$. For POD multiscale basis functions we use the columns of $\widetilde{B} = BV$. We have implemented the POD approaches using the L^2 inner product of the velocity. We observe an improvement of the approximation of the velocity and the saturation (cf. Section 5.3). For instance, we get a velocity error of 0.14% with POD and 0.09% without POD for an isotropic Gaussian distribution. For the saturation we have 2.02% with POD and 1.32% without POD. The gain in the POD approach is larger for the velocity since we consider the L^2 inner product of the velocity. It is not clear if this inner product is optimal for the quantity of interest: the water saturation. The choice of an optimal inner product for the saturation will be investigated in future work. For MLMC convergence we have not observed any gain using the POD approach. (cf. Section 5.6).

3.2 Local-solve-online ensemble level method (LSO)

The local-solve-online ensemble level method is similar to a reduced basis approach (e.g., [11, 21]). However, in comparison to reduced basis methods where the realizations fulfill special properties, we select the realizations for the pre-computations randomly. We apply the local-solve-online ensemble level method in the context of mixed multi-scale FEM as follows. The offline part coincided with the offline part of the above introduced NLSO method. We construct sets of basis functions based on a few (N_l) randomly chosen realizations of the coefficient $k_j(x) = k(x, \omega_j)$, $1 \leq j \leq N_l$ or we can use POD to select basis functions as described above.

After we have solved (2.6) for each realization we define for every edge $\iota = \{ii'\}$ a space $V_h^\iota := \bigoplus_{j=1}^{N_l} \Psi_{\iota,k_j}$. In this space V_h^ι we approximate in the online stage the solution $\tilde{w}_{\iota,\tilde{k}}$ of the auxiliary problem (2.6) with some local

boundary conditions with the coefficient $\tilde{k}(x) = k(x, \tilde{\omega})$ for a random $\tilde{\omega}$. As an approximation space for the velocity we define now

$$V_h^{LSO} = \bigoplus_{\iota \in \mathcal{I}} \{\tilde{\Psi}_{\iota,\tilde{k}}\},$$

for $\tilde{\Psi}_{\iota,\tilde{k}} = -\tilde{k}(x)\nabla \tilde{w}_{\iota,\tilde{k}}$.

For each computed multiscale velocity we solve the transport equation (2.2) to compute the saturation.

In this approach we use a different multiscale basis for each coefficient. That is the reason why the integrals $\int \tilde{\Psi}_{\iota,\tilde{k}} \tilde{\Psi}_{\iota',\tilde{k}}$ over a coarse block cannot be pre-computed. However, since each basis function $\tilde{\Psi}_{\iota,\tilde{k}}$ is a linear combination of the pre-computed basis functions Ψ_{ι,k_j}, $1 \leq j \leq N_l$, the calculations can be done inexpensively using pre-computed quantities.

As in the previous approach NLSO we can use POD to find the best local basis. Again we observe an improvement in the numerical simulations for approximating the velocity and the saturation (cf. Section 5.3). Analogous to the NLSO method the use of POD does not change the results for MLMC and MC significantly.

3.3 Comparison of LSO and NLSO

In contrast to the NLSO method, the online computations of the LSO method are divided into two parts, first one solves in each interior edge of the coarse region a local problem of size N_l^2 to get a basis for a chosen realization. Once multiscale basis functions are identified for each edge, the global problem is solved on a coarse grid by projecting the solution. In the NLSO approach we solve the equations on a N_l times larger coarse space (since we have N_l basis functions on each edge instead of one) while the accuracy of the method is not sacrificed at the expense of a specific boundary condition.

If the boundary condition g_ι in (2.6) uses the local information which does not depend on the permeability, the NLSO and the LSO approach have almost the same accuracy. In the NLSO case we approximate the velocity with

$$v = \sum_{\iota \in \mathcal{I}} \sum_{j=1}^{N_l} c_{\iota j} \Psi_{\iota,k_j}$$

and in the LSO approach with

$$v = \sum_{\iota \in \mathcal{I}} c_\iota \tilde{\Psi}_{\iota,\tilde{k}} = \sum_{\iota \in \mathcal{I}} c_\iota \sum_{j=1}^{N_l} c_j \Psi_{\iota,k_j}.$$

This is exactly the same if we have the freedom to choose c_j. Note that the coefficients c_j in LSO are determined from the solution of local problems that compute basis functions.

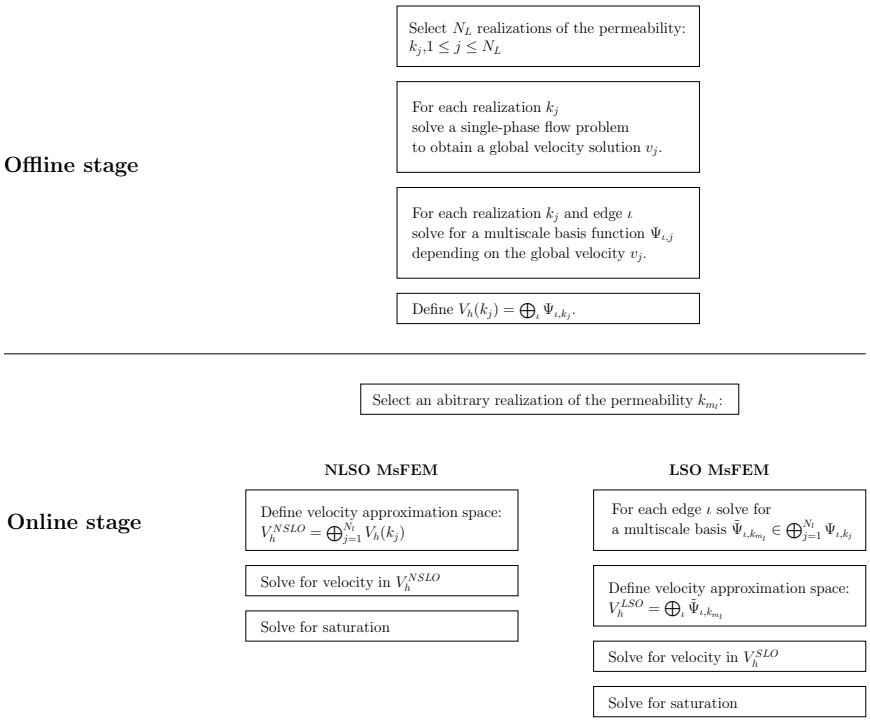


Figure 2: Comparison of LSO and NLSO.

Global information is important as it was demonstrated in [20, 12] theoretically and numerically. This is particularly true when the problem is solved multiple times. In this section, our goal is to show that when global information is used one cannot use LSO type approaches and need NLSO if an accurate solution is sought.

Since the cost of the online computation of the NLSO approach does not depend on the boundary condition g_ι it is reasonable to choose limited global boundary conditions. The first part of the online computations of the LSO method is to solve the auxiliary problem (2.6). Boundary conditions using global information would increase the computational cost of the method. With this choice of boundary condition there would be no computationally gain of an ensemble level method compared to a standard mixed MsFEM method for each particular realization in our setup. Therefore we choose local boundary conditions for the online part of the LSO method (global for the offline part). With this choice of boundary conditions the computational cost for NLSO and LSO is comparable. The NLSO approach is more accurate than the LSO method if global boundary conditions are used as we will show in Section 5.3.

4 Multi-level Monte Carlo methods using ensemble level mixed MsFEM

First we introduce multi-level Monte Carlo in a general way and after that we combine it with the introduced ensemble level mixed MsFEMs, NLSO and LSO. Denote G a generic random function, $G = G(x, \omega)$. We are interested in the efficient computation of the expectation of G, denoted by $E(G)$. The idea of multi-level Monte Carlo (MLMC) methods is to consider the quantity of interest at different levels l. We introduce levels smaller than L, such as $L-1$,..., 1 where we assume that the computation at the smallest level is cheap, while less accurate, to compute and $G_0 = 0$. Particularly, we assume $\|G_l - G\| \sim \frac{1}{N_l^\beta}$, with $N_1 \leq N_2 \leq \cdots \leq N_L$ and $\beta > 0$. N_l is related to the accuracy of the approximation (see discussion below). As an approximation of the expectation $E(G)$ we use the multi-level approximation

$$E^L(G_L) = \sum_{l=1}^{L} E_{M_l} \left(G_l - G_{l-1}\right), \tag{4.1}$$

where E_{M_l} denotes the arithmetic mean with M_l samples.

For error estimates, we consider the root mean square errors

$$\|E(G_L) - E^L(G_L)\| := \left(E(\|E(G_L) - E^L(G_L)\|_B^2)\right)^{\frac{1}{2}} \tag{4.2}$$

with an appropriate norm $\|\cdot\|_B$. We get

$$\begin{aligned}
& \|E(G_L) - E^L(G_L)\| \\
= \; & \|E\left(\sum_{l=1}^{L}(G_l - G_{l-1})\right) - \sum_{l=1}^{L} E_{M_l}(G_l - G_{l-1})\| \\
\leq \; & \sum_{l=1}^{L}\|(E - E_{M_l})(G_l - G_{l-1})\| \\
\leq \; & \sum_{l=1}^{L}\frac{1}{\sqrt{M_l}}\|G_l - G_{l-1} - E(G_l - G_{l-1})\| \\
\lesssim \; & \sum_{l=1}^{L}\frac{1}{\sqrt{M_l}}\|G - G_l\| + \frac{1}{\sqrt{M_1}} \\
\lesssim \; & \sum_{l=2}^{L}\frac{1}{\sqrt{M_l}}\frac{1}{N_l^{\beta}} + \frac{1}{\sqrt{M_1}}.
\end{aligned}$$

To equate the error terms we choose

$$M_l = C\begin{cases} N_L^{2\beta}, & l = 1 \\ \left(\frac{N_L}{N_l}\right)^{2\beta}, & 2 \leq l \leq L, \end{cases} \tag{4.3}$$

then we end with

$$\|E(G_L) - E^L(G_L)\| = O\left(\frac{1}{N_L^{\beta}}\right). \tag{4.4}$$

We are interested in predicting of the saturation field and will measure the mean square error in the saturation field. We use both ensemble level mixed MsFEMs -NLSO and LSO- to calculate G_l where N_l realizations of the permeability field are chosen to compute the basis for the whole ensemble.

At each level l we select a different number of realizations of the permeability field N_l, $N_1 \leq N_2 \leq \cdots \leq N_L$ to build a low dimensional approximation space for velocity that captures both small scale (sub coarse-grid) spatial variability in the permeability data and stochastic variability due to uncertainties in the data. In particular, we calculate the velocity at level l for M_l realizations to get saturations $S_{l,m}$ with $1 \leq l \leq L$ and $1 \leq m \leq M_l$ to build the MLMC approximation of the expected value of the fine scale saturation. For clarity we summarize the basic steps below.

1. *Generation of coarse grid.*

   - Partition the domain into a coarse grid. The coarse grid is a partitioning of the fine grid where each cell in the fine grid belongs to a unique block in the coarse grid and each coarse grid block is connected. [1].

2. *Construction of multiscale approximation space V_h:*

   - Select N_L realizations from the stochastic permeability distribution.
   - For each selected realization $1 \leq j \leq N_L$:
     - Solve (2.4) on the fine grid using a suitable mass conservative numerical method to obtain a 'global' velocity solution v_j.
     - Compute the multiscale basis functions: For each edge Γ_ι, $\iota = \{ii'\}$, set
     $$g_\iota(k_j) = \frac{v_j \cdot n_{ii'}}{\int_{\Gamma_\iota} v_j \cdot n_{ii'} ds},$$
     and solve (2.6) to obtain w_{ι,k_j} and subsequently Ψ_{ι,k_j}.
     - Define $V_h(k_j) = \bigoplus_\iota \Psi_{\iota,k_j}$.

3. *Multi-level mixed MsFEM computations for estimating an expectation at level l, $1 \leq l \leq L$:*

   - Select M_l realizations of the permeability k_{m_l}, $1 \leq m_l \leq M_l$.
   - **LSO method**:
     - Compute: $\tilde{\Psi}_{\iota,k_{m_l}} \in V_h^\iota = \bigoplus_{j=1}^{N_l} \Psi_{\iota,k_j}$ by solving (2.6) with k_{m_l} and local B.C..
     - Define $V_h^{LSO} = \bigoplus_\iota \{\tilde{\Psi}_{\iota,k_{m_l}}\}$
     - Solve two-phase flow and transport (2.1)-(2.2) for $S_{l,m}$. At each time step, the velocity field is constructed by solving (2.1) on a coarse grid using V_h^{LSO}.
   - **NLSO method**:
     - Define $V_h^{NLSO} = \bigoplus_{j=1}^{N_l} V_h(k_j)$.
     - Solve two-phase flow and transport (2.1)-(2.2) for $S_{l,m}$. At each time step, the velocity field is constructed by solving (2.1) on a coarse grid using V_h^{NLSO}.
   - Calculate the arithmetic mean
   $$E_{M_l}(S_l - S_{l-1}) = \frac{1}{M_l} \sum_{m=1}^{M_l} (S_{l,m} - S_{l-1,m}). \tag{4.5}$$

4. *MLMC approximation of $E(S)$*

$$E^L(S_L) = \sum_{l=1}^{L} E_{M_l}(S_l - S_{l-1}). \tag{4.6}$$

Next, we discuss the work for MLMC and compare it with MC. We will consider two cases: single-phase flow ($\lambda(S) = 1$ and $f(S)$ nonlinear) and two-phase flow. In both cases, we will compare the saturation field at a certain time instant. In all simulations, we will ignore the cost of pre-computations as

basis functions are fixed throughout the simulations. Note that we have almost the same numerical costs for NLSO and for LSO, if we assume we solve the pressure equation optimally. The computational cost for solving the pressure equation each time instant with N_l basis functions (at level l) for a coarse grid H_j is $2\left(H_j^{-1}-1\right)N_l^2H_j^{-1}$. For LSO we solve at each interior edge an N_l^2 problem and solve the pressure equation for only one set of basis functions we get $2\left(H_j^{-1}-1\right)\left(N_l^2+1\right)H_j^{-1}$. In the following we neglect the costs of solving the pressure equation for one set of basis functions in LSO, such that we end with the same numerical costs for NLSO and LSO. But of course it is possible to calculate the online multiscale LSO basis of each edge in parallel, such that the computational time for LSO is much smaller. In our simulations, we will equate the cost of solving the pressure equation for MC and MLMC and compare the accuracy of these approximations. Although for single-phase flow this comparison is accurate (up to the cost of computation of basis functions), one needs to take into account the cost of solving the saturation equation in two-phase flow and transport simulations. The cost of the computation for the saturation equation at each time instant is the same at any level because we use a coarse-grid velocity field. We note the cost of solving the pressure equation is larger than that for the saturation equation on a coarse grid because the convergence of iterative solvers requires many iterations for multiscale problems and there are more degrees of freedom. Since we use several basis functions per coarse edge in the ensemble level multiscale method, the cost computing the pressure solution can be several times larger than that of the saturation equation because the coarse system is several times larger for the pressure equation. We will ignore the cost of saturation computation on a coarse grid in two-phase flow examples.

We have the following computational costs for MLMC based on pressure:

$$\begin{aligned} & W_{MLMC} \\ = & \sum_{l=1}^{L}\left(2\left(H_j^{-1}-1\right)N_l^2H_j^{-1}\right)M_l \\ = & 2N_L^{2\beta}H_j^{-1}(H_j^{-1}-1)\left(\sum_{l=2}^{L}N_l^{2-2\beta}+N_1^2\right). \end{aligned}$$

We compare the costs with the computational costs for MC

$$W_{MC}=\left(2\left(H_j^{-1}-1\right)\widehat{N}^2H_j^{-1}\right)\widehat{M}. \tag{4.7}$$

In our numerical simulations, we will equate the work and compare the accuracy of MLMC and MC. When comparing the work, we will calculate the number of realizations for detailed two-phase flow and transport simulations that are needed.

We have observed that the accuracy of ensemble level multiscale methods increases as we increase the dimension of the coarse space (see [3] and the discussions below). However, this accuracy can not be estimated, in general.

We propose the use of an empirical procedure that allows, based on simulations with a few samples, to estimate the convergence rate of ensemble level mixed MsFEM; further, using these estimates we select the number of realizations, M_i (see (4.3)) in MLMC.

5 Numerical results

In our numerical examples, we will consider permeability fields described by two-point correlation functions. We will use the Karhunen-Loève expansion (KLE) to parameterize these permeability fields and apply the MLMC algorithm as described above. We will compute the number of MC realizations needed to achieve the same amount of work and compare the accuracy of MLMC and MC. First, we briefly describe the permeability parameterization and then we will present numerical results.

5.1 Permeability parameterization

To obtain the permeability field in terms of an optimal L^2 basis, we use the Karhunen-Loève expansion (KLE) [19]. For our numerical tests, we truncate the expansion and represent the permeability matrix by a small number of random parameters. We briefly recall the facts of the KLE. Consider $Y(x,\omega) = \log[k(x,\omega)]$, where ω represents randomness. We assume that $E[Y(x,\omega)] = 0$ and $R(x,y) = E\left[Y(x)Y(y)\right]$. We will expand $Y(x,\omega)$ as

$$Y(x,\omega) = \sum_{k=1}^{\infty} Y_k(\omega)\psi_k(x), \qquad\qquad Y_k(\omega) = \int_\Omega Y(x,\omega)\psi_k(x)dx,$$

where $\psi_k(x)$ is an orthonormal basis that are eigenvectors of $R(x,y)$ when discretized. More precisely, $\{\psi_k\}$ is a complete basis in $L^2(\Omega)$. It follows that $\psi_k(x)$ are eigenfunctions of $R(x,y)$:

$$\int_\Omega R(x,y)\psi_k(y)dy = \lambda_k\psi_k(x), \qquad k = 1,2,\ldots, \tag{5.1}$$

where $\lambda_k = E[Y_k^2] > 0$. Note that $E(Y_iY_j) = 0$ for all $i \neq j$. Denoting $\eta_k = Y_k/\sqrt{\lambda_k}$ ($E(\eta_k) = 0$ and $E(\eta_i\eta_j) = \delta_{ij}$), we have

$$Y(x,\omega) = \sum_{k=1}^{\infty} \sqrt{\lambda_k}\eta_k(\omega)\psi_k(x), \tag{5.2}$$

where ψ_k and λ_k satisfy (5.1). The randomness is represented by the scalar random variables η_k. After the discretization of the domain Ω in a rectangular mesh, we truncate the KLE (5.2) to finite terms. We only keep the leading order terms (quantified by the magnitude of λ_k) and capture most of the energy of the stochastic process $Y(x,\omega)$. For an N-term KLE approximation $Y_N = \sum_{k=1}^{N} \sqrt{\lambda_k}\eta_k\psi_k$, the energy ratio of the approximation is defined as $e(N) :=$

$\frac{E\|Y_N\|^2}{E\|Y\|^2} = \frac{\sum_{k=1}^{N}\lambda_k}{\sum_{k=1}^{\infty}\lambda_k}$. If $\lambda_k, k = 1, 2, \ldots$, decay very fast, then the truncated KLE would be a good approximation of the stochastic process in the L^2 sense.

We will consider two types of correlation functions. The first one is log-Gaussian

$$R(x, y) = \sigma^2 \exp\Big(-\frac{|x_1 - y_1|^2}{2l_1^2} - \frac{|x_2 - y_2|^2}{2l_2^2}\Big), \tag{5.3}$$

and the next one is log-Exponential

$$R(x, y) = \sigma^2 \exp\Big(-\frac{|x_1 - y_1|}{l_1} - \frac{|x_2 - y_2|}{l_2}\Big). \tag{5.4}$$

In the above formula, l_1 and l_2 are the correlation lengths in each dimension, and $\sigma^2 = E(Y^2)$ is a constant that represents the variance of the permeability field. In the first case, we expect faster decay of eigenvalues compared to the second case, log-Exponential, for a given set of correlation lengths.

5.2 Experimental setup

In our simulations we consider the traditional quarter-of-a-five-spot with no-flow boundary conditions. The domain Ω is a square and we inject water at the upper left corner while the producer is placed at the lower right one. The pressure equation (2.1) is solved with mixed MsFEM and we solve for the saturation with an implicit scheme. In the numerics we solve (2.1) on a coarse grid or on a fine grid. If we solve on the fine grid, we take the transport equation with the right-hand side zero. On the fine grid we solve a single-phase flow problem and on the coarse grid a two-phase flow problem. In both cases the permeability field $Y(x)$ is given on a 100×100 fine Cartesian grid. As mentioned above, we consider Gaussian and Exponential covariance functions. In all examples we have $\sigma^2 = 2$. In particular we consider:

- Isotropic Gaussian field: correlation length $l_1 = l_2 = 0.2$, stochastic dimension 10
- Anisotropic Gaussian field: correlation length $l_1 = 0.5$ and $l_2 = 0.1$, stochastic dimension 12
- Isotropic Exponential field: correlation length $l_1 = l_2 = 0.2$, stochastic dimension 300
- Anisotropic Exponential field: correlation length $l_1 = 0.5$ and $l_2 = 0.1$, stochastic dimension 350

To build the multiscale basis, we generate N_L independent realizations of the permeability.

We have also selected a POD basis as described in Section 3. But we have not observed any gain for MLMC. For this reason we mostly show the results for randomly chosen realization for the offline stage. We briefly show some results using the POD approach to pre-compute the multiscale basis for two-phase flow.

The multiscale basis functions are not recomputed during the simulations, i.e., they are computed at time zero. We compare the MLMC accuracy of the saturation at a certain time instance (PVI= 0.8) with the accuracy of standard MC at level L with the same amount of costs; therefore, we choose

$$\widehat{M} = \frac{\sum_{l=1}^{L} N_l^2 M_l}{N_L^2}. \tag{5.5}$$

As reference, we use the arithmetic mean of M_{ref} samples of the saturation solved with the multiscale velocity where the basis is calculated for the permeability realization which is used in the pressure equation. We denote the reference saturation with S_{ref}. We consider the square root of the arithmetic mean of square of the relative L^2 errors, e.g., for MLMC we consider

$$\text{MLMC error} = \sqrt{\frac{1}{J}\sum_{j=1}^{J} \frac{\|S_{ref} - S^j_{MLMC}\|^2_{L^2}}{\|S_{ref}\|^2_{L^2}}}, \tag{5.6}$$

where S^j_{MLMC} denotes the MLMC approximation of the expectation for a given set of permeability realization $\{k_1, \cdots, k_{N_L}\}$ to compute the multiscale basis functions (cf. (2.6)) and different realizations $\{k_1, \cdots, k_{m_1}\}$ to solve the flow an transport equations (cf. (2.1)-(2.2)) to compute the saturation.

5.3 Comparison of the NLSO and LSO approach for single-phase flow

In this section we briefly study the differences of the two ensemble level mixed MsFEM and the influence of the choice of different boundary conditions using local or limited global information for single-phase flow.

We show that methods using local boundary conditions have a residual error no matter how many basis functions we pick.

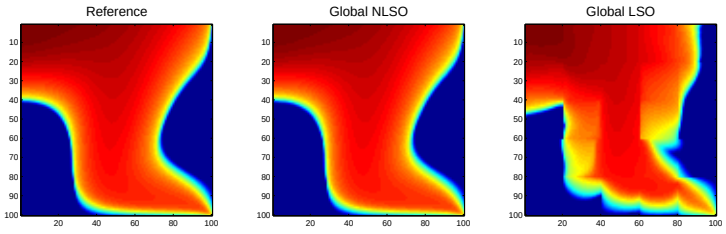
Therefor we pre-compute 12 basis functions with either local or global boundary conditions. We study the influence based on an isotropic Gaussian and an anisotropic Exponential distribution. For the selected realization k_j we consider the L^2-error between the water saturation computed with one of the ensemble level methods with either local or global boundary conditions and a reference saturation. As reference we use either the fine-scale water saturation, S^j_{ref}, or the multiscale saturation, S^j_{refloc}, where the basis is calculated for the realization k_j with local boundary conditions. In our case the local boundary conditions are a combination of the permeability in different cells. Note, that the goal is not to compare local and global boundary conditions but the goal is to compare the NLSO and LSO approaches. We show that the LSO method does not remove the residual error. In Table 1 we computed the mean L^2-errors of 100 different realizations for all the combinations. Again, note that we solve the online problem for LSO with local boundary conditions. The mean error between the reference saturation with local and global boundary conditions is

	isotropic Gaussian				anisotropic Exponential			
	gNLSO	gLSO	lNLSO	lLSO	gNLSO	gLSO	lNLSO	lLSO
$\|S_{ref} - S\|$	0.33	16.06	1.91	16.16	16.18	33.60	21.47	34.00
$\|S_{refloc} - S\|$	16.21	1.88	16.02	0.75	30.18	19.88	27.19	17.59

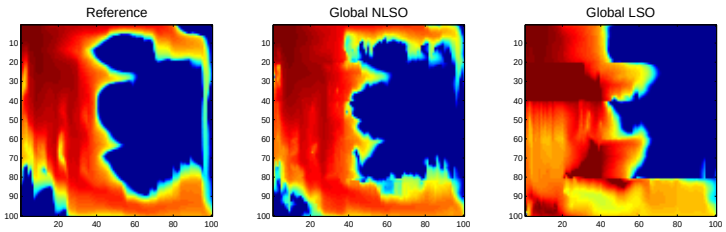
Table 1: Mean errors (percent) of 100 realizations for the different methods and boundary conditions for the single-phase flow example.

approximately 16% for the Gaussian and 34% for the Exponential distribution. It is known from literature (cf. [4]) that the source of the errors are the local boundary conditions and boundary conditions using global information should be used to overcome this problem.

From the table we see that in the LSO method we introduce an additional error by solving the online problem with local boundary conditions. The NLSO seems to be a good approximation of the fine-scale saturation, while the solution of LSO is closer to the local reference. For instance, we have an error of less than 1% for the NLSO with global boundary conditions for the Gaussian distribution. The error of the LSO approach to the global reference is of the same size as the error between the global reference and the local one independent of the underlying distribution. In both approaches global boundary conditions give the better approximation of the fine-scale saturation. For this reason we will consider only global boundary conditions in the following. In Figure 3 we show the water saturations with global boundary conditions for one sample of the isotropic Gaussian and the anisotropic Exponential random field.



(a) Isotropic Gaussian



(b) Anisotropic Exponential

Figure 3: Water saturation for global boundary condition for the different methods.

5.4 Convergence rate

In Section 4 we assume $\|G_l - G\| = C\frac{1}{N_l^\beta} =: \delta_l$, with $\delta_1 > \delta_2 > \cdots \delta_L$. In this section we investigate δ_l where G denotes the saturation with the basis calculated for the permeability realization which is used in the pressure equation and G_l the saturation with a pre-computed basis for N_l permeability realizations. With these δ_ls, it is possible to find appropriate choices of realizations M_l at each level, namely

$$M_l = C \begin{cases} \left(\frac{1}{\delta_L}\right)^2 (\mathrm{std}(G) + \delta_1^2), & l = 1 \\ \left(\frac{\delta_l}{\delta_L}\right)^2, & 2 \leq l \leq L. \end{cases}$$

In the previous section we observed that the LSO approach does not converge to the fine scale saturation. For this reason we consider the NLSO case only. Furthermore, we observe similar convergence rates if we choose the realization for the pre-computations randomly or if we use the POD approach. So we present the results for the random selection case. For further studies we refer to [17].

To determine the convergence rate, we choose $\mathcal{N} = (3, 6, 12)$ and calculate the mean over $M = 100$ permeability realizations as follows. For 10 sets of permeability realizations we compute the multiscale basis and each of these sets we use to compute the error for 10 different permeability realizations. We compute the arithmetic mean of the 10×10 numbers. The resulting δs depend on the underlying distribution of the covariance function, but at least in the two-phase flow example the ratio $\frac{\delta_l}{\delta_L}$ remains almost the same for all distributions. For the Gaussian distributions (isotropic and anisotropic) we get approximately $(2.7, 1.6, 1)$ and for the Exponential ones $(2.0, 1.5, 1)$. In the single-flow case the ratio is the same for the isotropic and anisotropic Exponential distribution we get for the ratios $(1.6, 1.3, 1)$. If the covariance is Gaussian, the decay is much faster than in the Exponential case and the decay is even much faster for the isotropic Gaussian distribution. The ratios in the isotropic Gaussian case are $(13.6, 5.1, 1)$ and in the anisotropic Gaussian $(5.2, 2.6, 1)$.

5.5 Single-phase flow

As mentioned above, we denote the example as single-phase flow, if the total mobility $\lambda(S) = 1$. We take the flux term $f(S) = \frac{S^2}{S^2+(1-S)^2}$ (see (2.2)). Since the mobility does not depend on the saturation, we solve (2.1) only once and update the transport equation for each time step.

The parameters we use for our simulation and the resulting relative errors for the different distributions are summarized in Table 2. In Figure 4 we have plotted the water saturation for the different covariance functions (isotropic and anisotropic Gaussian, isotropic and anisotropic Exponential) and the different methods (MLMC and MC) in the case of NLSO. In Figure 5 we used LSO. We choose the numbers of samples at each level as $\mathfrak{M} = (70, 20, 10)$ and the

	isotropic Gaussian	anisotropic Gaussian	isotropic Exponential	anisotropic Exponential
(N_1, N_2, N_3)	$(3, 6, 12)$	$(3, 6, 12)$	$(24, 48, 96)$	$(24, 48, 96)$
(M_1, M_2, M_3)	$(70, 20, 10)$	$(70, 20, 10)$	$(70, 20, 10)$	$(70, 20, 10)$
$\widehat{N}$	12	12	96	96
$\widehat{M}$	20	20	20	20
M_{MCref}	500	500	500	500
MLMC error NLSO	0.0758	0.0847	0.0934	0.0889
MC error NLSO	0.1439	0.1222	0.1389	0.1343
$\frac{\text{MC error}}{\text{MLMC error}}$ NLSO	1.90	1.44	1.49	1.51
MLMC error LSO	0.1101	0.1189	0.1781	0.1711
MC error LSO	0.3027	0.4244	0.2112	0.1839
$\frac{\text{MC error}}{\text{MLMC error}}$ LSO	2.75	3.57	1.19	1.07

Table 2: Parameters and errors for the single-phase flow example.

dimension of the approximation space (defined as number of independent samples selected to construct the multiscale space) for the Exponential distributions eight times larger than for the Gaussian tests. We equate the computational costs and compare the resulting relative errors for MLMC and MC. With this choice of realizations for MLMC, we get for MC with equated costs $\widehat{M} = 20$, where $\widehat{M}$ is the number of permeability realizations needed for forward simulations. We repeat these calculation for 20 sets of multiscale basis function, i.e., $J = 20$ in (5.6).

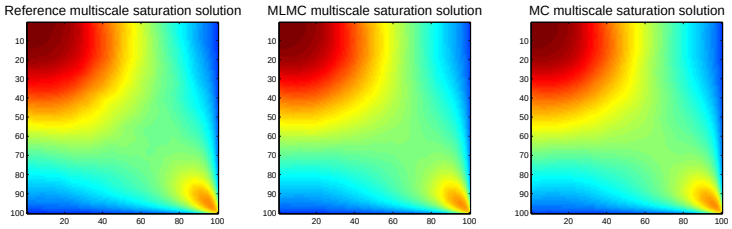
In the NLSO case we get for MLMC relative errors of approximately 9 percent and for MC approximately 13 percent. This result is almost independent of the underlying distribution, but note that for the Exponential test cases the computational work is eight times larger than for the Gaussian ones. So the MC error is 1.5 times larger than the MLMC one. In the LSO case we introduce an additional error by projecting the multiscale basis onto the space of the N_l pre-computed basis functions due to the local boundary conditions. So both errors are larger, but the MC error is about 3 times larger than the MLMC one.

We observe similar results if the POD approach is used. So we show the results for the random case. We will only show some results for the POD case in the two-phase flow example (cf. Section 5.6).

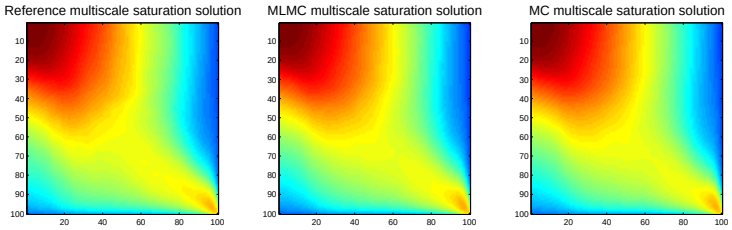
Again we note that one could do the computations of the multiscale basis of the LSO method in parallel and so it is possible to decrease the errors by keeping the computational time fixed.

5.6 Two-phase flow

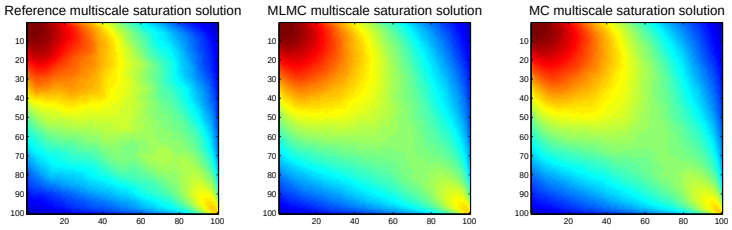
We consider a 5×5 coarse grid, such that every grid block contains a 20×20 cell partition from the fine grid. We choose $\mu_w = 1$, $\mu_0 = 1$, $k_{rw}(S) = S^2$ and $k_{ro}(S) = (1 - S)^2$. We solve the pressure equation on the 5×5 coarse grid with mixed MsFEM for each time step. The parameters we use for our simulation and the resulting relative errors for the different covariance functions (isotropic and anisotropic Gaussian, isotropic and anisotropic Exponential) are summarized in Table 3. In Figure 6 we have plotted the water saturation for the different distributions and the different methods (MLMC and MC) in the case of NLSO and in Figure 7 we used LSO In all tests we use the same number of realizations



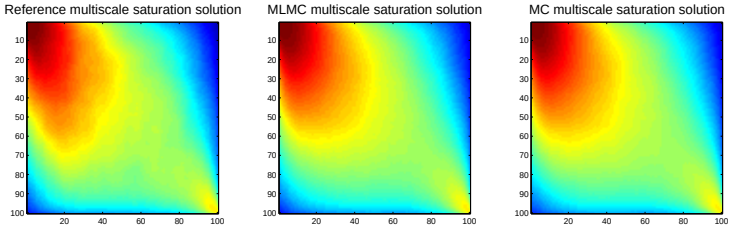
(a) Isotropic Gaussian



(b) Anisotropic Gaussian

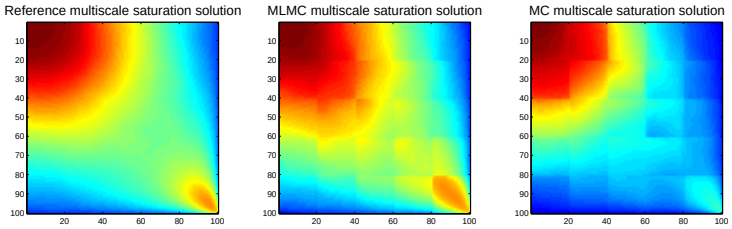


(c) Isotropic Exponential

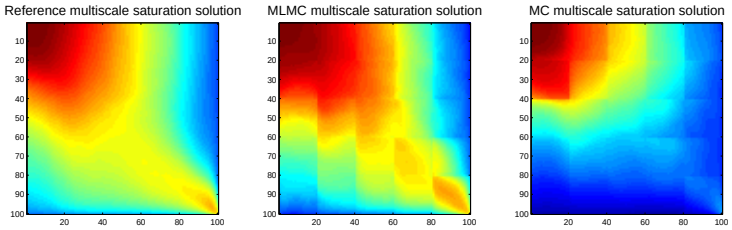


(d) Anisotropic Exponential

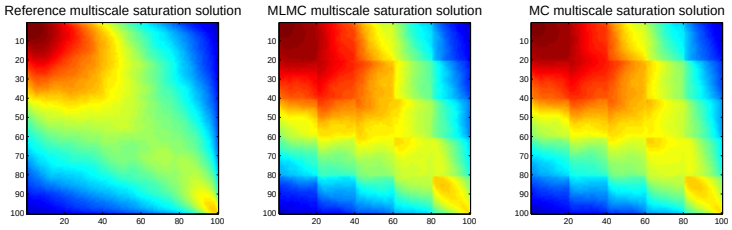
Figure 4: Water saturation for the different methods and distributions for single-phase flow using NLSO



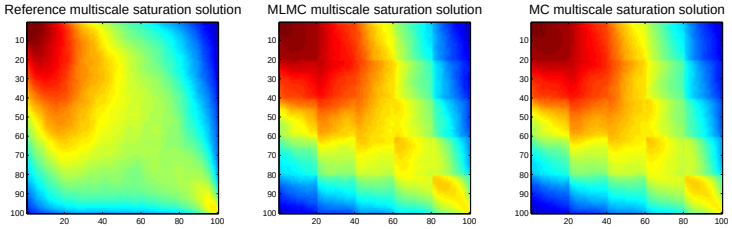
(a) Isotropic Gaussian



(b) Anisotropic Gaussian



(c) Isotropic Exponential



(d) Anisotropic Exponential

Figure 5: Water saturation for the different methods and distributions for single-phase flow using LSO.

	isotropic Gaussian	anisotropic Gaussian	isotropic Exponential	anisotropic Exponential
(N_1, N_2, N_3)	$(3, 6, 12)$	$(3, 6, 12)$	$(24, 48, 96)$	$(24, 48, 96)$
(M_1, M_2, M_3)	$(70, 20, 10)$	$(70, 20, 10)$	$(70, 20, 10)$	$(70, 20, 10)$
$\widehat{N}$	12	12	96	96
$\widehat{M}$	20	20	20	20
M_{MCref}	500	500	500	500
MLMC error NLSO	0.0522	0.0497	0.0529	0.0543
MC error NLSO	0.0989	0.0849	0.0947	0.0898
$\frac{\text{MC error}}{\text{MLMC error}}$ NLSO	1.90	1.71	1.79	1.65
MLMC error LSO	0.0448	0.0854	0.0581	0.1161
MC error LSO	0.0839	0.1056	0.1039	0.1274
$\frac{\text{MC error}}{\text{MLMC error}}$ LSO	1.72	1.24	1.79	1.10
MLMC error POD NLSO	0.0479	0.0563	0.0568	0.0583
MC error POD NLSO	0.0884	0.0937	0.0972	0.0911
$\frac{\text{MC error}}{\text{MLMC error}}$ LSO	1.85	1.67	1.71	1.56
MLMC error POD LSO	0.0565	0.0843	0.0652	0.1299
MC error POD LSO	0.0992	0.1083	0.0952	0.1524
$\frac{\text{MC error}}{\text{MLMC error}}$ LSO	1.76	1.29	1.46	1.17

Table 3: Parameters and errors for the two-phase flow example.

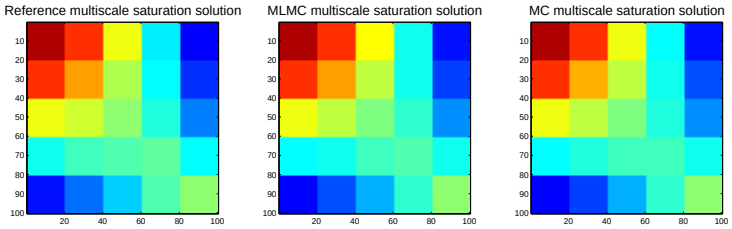
for MLMC and following the same number of realizations for MC with equated costs. Namely $\mathfrak{M} = (70, 20, 10)$ and $\widehat{M} = 20$. But the number of basis functions differs for the Gaussian and Exponential cases. For the Exponential test, the dimension of each level is eight times larger than for the Gaussian. This means we increase the costs by eight. As in the single-phase example we choose $J = 20$ in (5.6).

If we use NLSO the MC error is approximately 1.75 times larger than the MLMC error. Note that the gain is slightly larger for the isotropic distributions than for the anisotropic ones. For all distributions we end with a relative MLMC error of approximately 5 and a MC error of 9 percent. For LSO the errors are comparable. However, in this example we find that the gain is larger for isotropic distributions in comparison to the anisotropic ones. In the isotropic cases the MC error is about 1.75 times larger and for the anisotropic ones 1.2 times larger. Independent of the method -NLSO or LSO- we expect the ratio to increase if more levels are used.

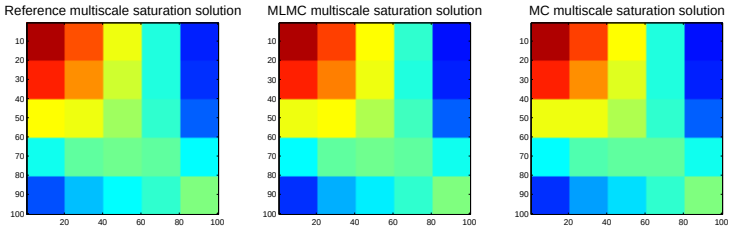
Next we briefly mention POD results and show the convergence rate does not improve. As you can see in Table 3 some of the errors decrease, e.g., for NLSO with Gaussian distribution, and some increase, e.g., NLSO with anisotropic Gaussian. For NLSO the ratio of the MC and the MLMC error decreases slightly if we use POD for the offline computations of the basis functions. For LSO in most cases these ratios increase. However, since there is no clear trend for the errors, we see no gain in using the POD approach for MLMC.

6 Conclusions

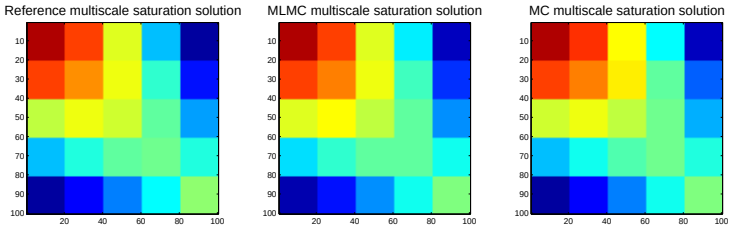
In this paper, we combine multiscale finite element methods -NLSO and LSO- and multi-level Monte Carlo (MLMC) techniques to speed-up Monte Carlo simulations. The multiscale methods solve the flow equation on a coarse grid with pre-computed basis functions using a few members of the ensemble. These basis functions are used to approximate the solution on a coarse grid for an arbitrary



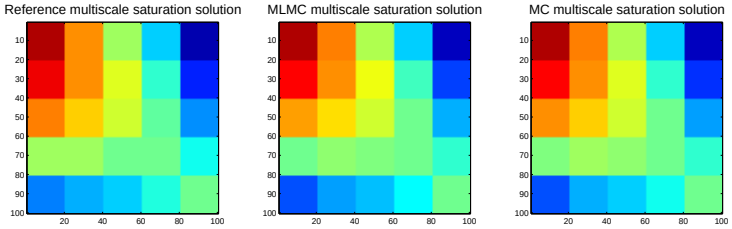
(a) Isotropic Gaussian



(b) Anisotropic Gaussian

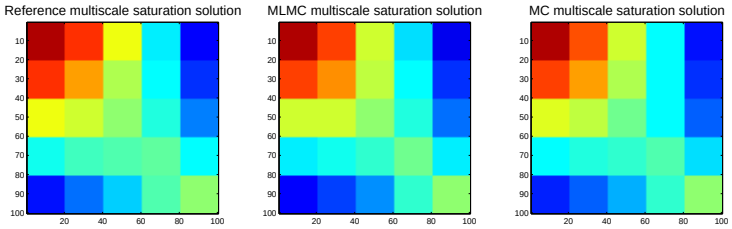


(c) Isotropic Exponential

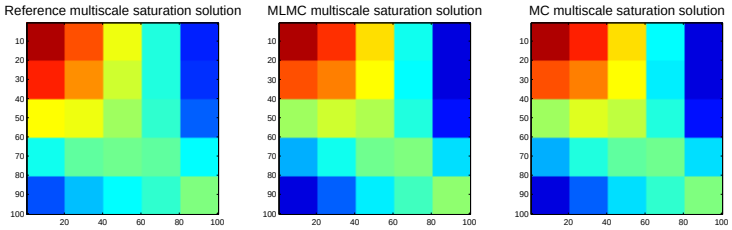


(d) Anisotropic Exponential

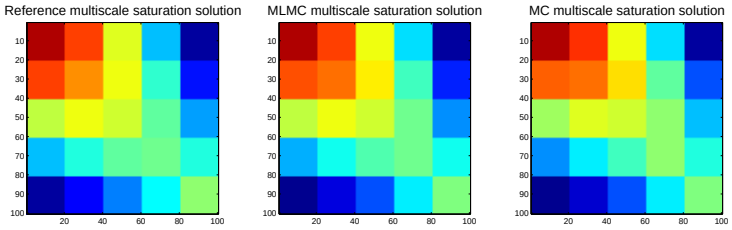
Figure 6: Water saturation for the different methods and distributions for two-phase flow using NSLO.



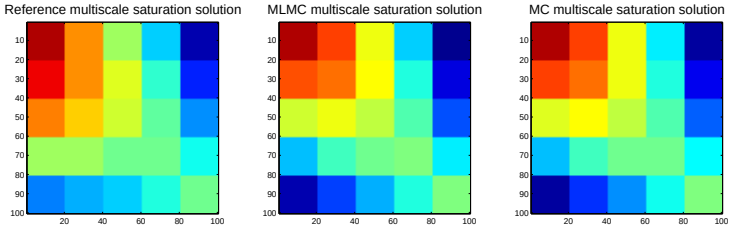
(a) Isotropic Gaussian



(b) Anisotropic Gaussian



(c) Isotropic Exponential



(d) Anisotropic Exponential

Figure 7: Water saturation for the different methods and distributions for two-phase flow using LSO.

realization. The use of larger dimensional coarse spaces yields more accurate solutions. In multi-level Monte Carlo methods ([14, 13]) more accurate (and expensive) forward simulations are run with a fewer samples while less accurate (and inexpensive) forward simulations are run with a larger number of samples. Selecting the number of expensive and inexpensive simulations carefully, one can show that MLMC can provide a better accuracy at the same cost as MC. In our simulations, we use various dimensional coarse spaces and perform a number of forward simulations at different resolutions. These simulations results are further used within the MLMC framework to speed-up MC calculations. We present basic aspects of the algorithm and numerical results for coupled flow and transport in heterogeneous porous media.

7 Acknowledgment

This work was partially support by the DFG project. The implementation of the mixed MsFEM is based on the code of Aarnes. For further details we refer to [5, 18].

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