MIMO state space models of piezomechanical systems with exact impedance mapping

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IV European Conference on Computational Mechanics
Palais des Congrès, Paris, France, May 16-21, 2010

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MIMO state space models of piezomechanical systems with exact impedance mapping

Outline

1. Motivation
2. State space model from superelement matrices
3. State space model from eigenfrequencies and eigenvectors
4. Numerical example
5. Conclusions

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Motivation

Application

- of piezo sensors and actuators to influence vibration:
  - active vibration control
  - shunt damping

Simulation

- to design the control strategy
- to design electrical devices

Integration

- in system simulation
- interface of FEM and system simulation

Focus

- practical application of simulation methods and models
- based on commercial codes (ANSYS®, Matlab®)

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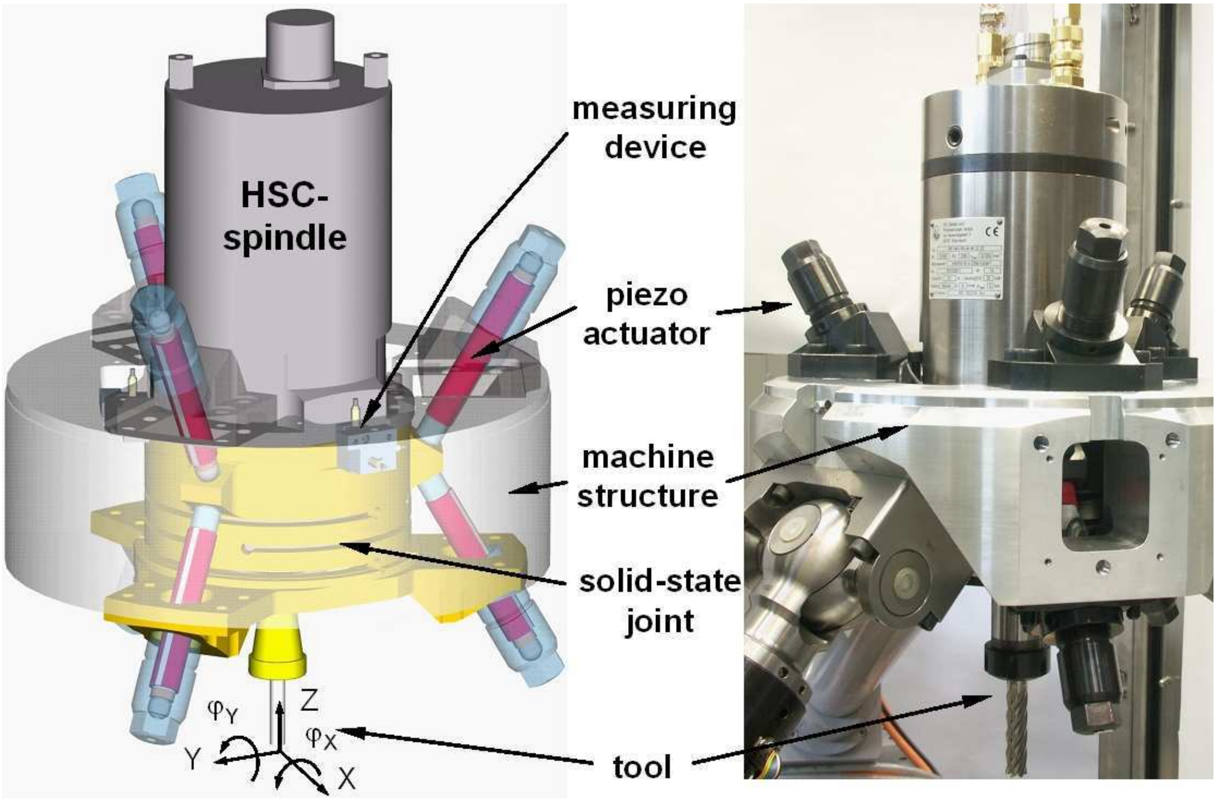
Motivation

Active spindle support
with piezo stack actuators



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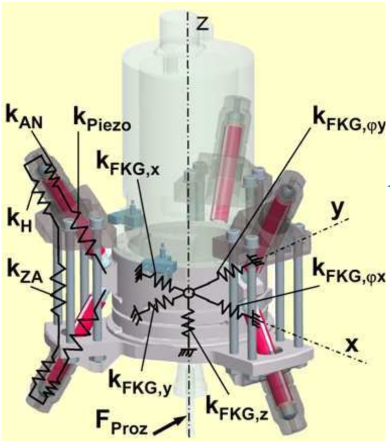
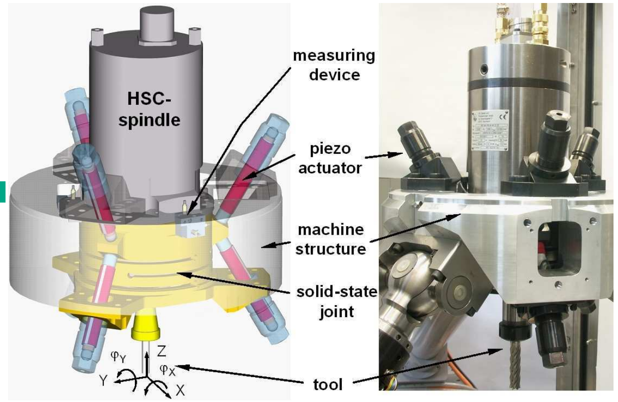
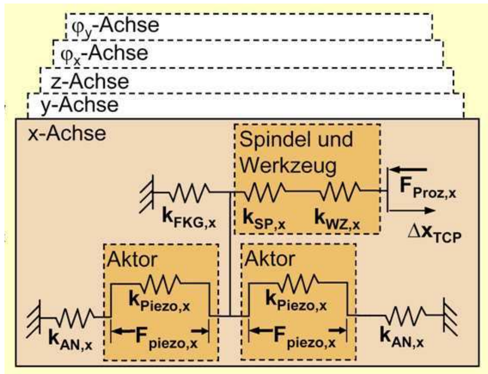
Motivation





- System description by concentrated parameters
- Only for simple coupling of DOF
- Only for certain multivariable systems



- Input/output transfer functions

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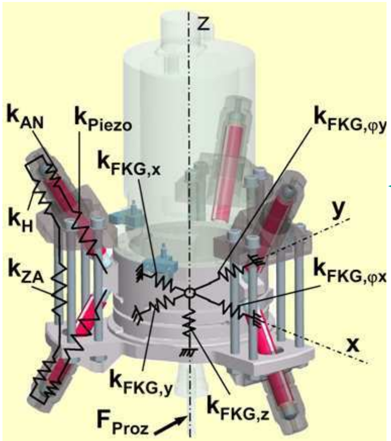
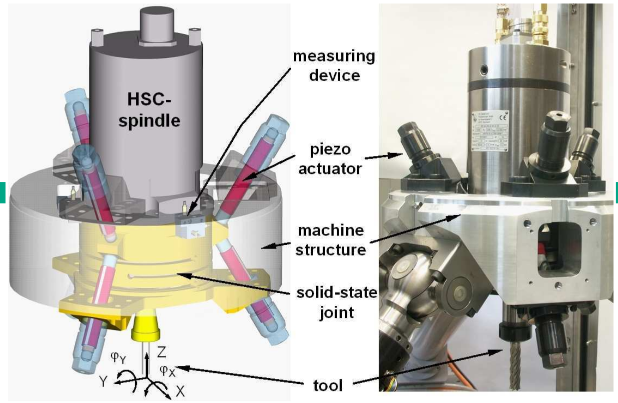
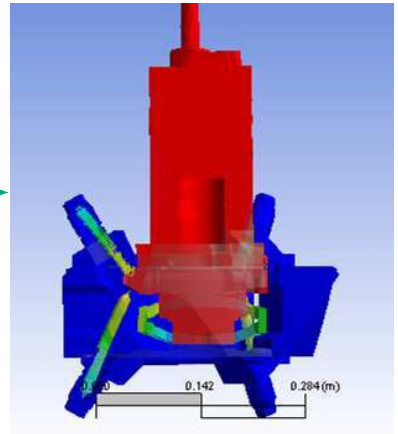
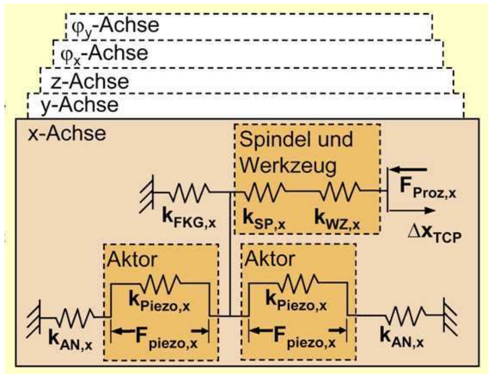
Motivation





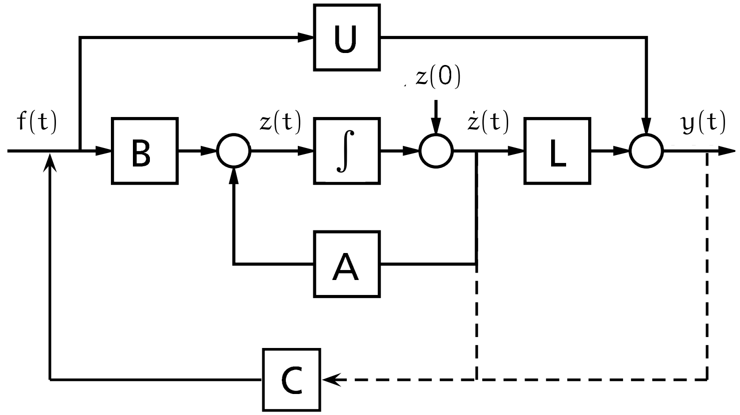


- System description by concentrated parameters
- Only for simple coupling of DOF
- Only for certain multivariable systems



- Input/output transfer functions

- System description by finite element simulation
- For locally distributed actuation
- For multivariable systems



- State space representation

State space model of mechanical systems

Discretized equation of motion with a large number of DOF

$$M \cdot \ddot{u} + D \cdot \dot{u} + K_{uu} \cdot u = F$$

General state space model with a limited number of input and output signals

$$\dot{z} = A \cdot z + B \cdot f \qquad y = L \cdot z + U \cdot f$$

State variables

$$z = \begin{pmatrix} u \\ \dot{u} \end{pmatrix}$$

State equation

$$\begin{pmatrix} \dot{u} \\ \ddot{u} \end{pmatrix} = \begin{pmatrix} 0 & I \\ -M^{-1} \cdot K_{uu} & -M^{-1} \cdot D \end{pmatrix} \begin{pmatrix} u \\ \dot{u} \end{pmatrix} + \begin{pmatrix} 0 \\ M^{-1} \end{pmatrix} F$$

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State space model of piezo-mechanical systems

Discretized equation of motion for piezo-mechanical systems with a large number of DOF

$$\begin{pmatrix} M & 0 \\ 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} \ddot{u} \\ \ddot{\Phi} \end{pmatrix} + \begin{pmatrix} D & 0 \\ 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} \dot{u} \\ \dot{\Phi} \end{pmatrix} + \begin{pmatrix} K_{uu} & K_{u\Phi} \\ K_{u\Phi}^T & K_{\Phi\Phi} \end{pmatrix} \cdot \begin{pmatrix} u \\ \Phi \end{pmatrix} = \begin{pmatrix} F \\ Q \end{pmatrix}$$

$$\Downarrow$$

General state space model with a limited number of input and output signals

$$\dot{z} = A \cdot z + B \cdot f \qquad y = L \cdot z + U \cdot f$$

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General state space model with a limited number of input and output signals

$$\dot{z} = A \cdot z + B \cdot f \qquad y = L \cdot z + U \cdot f$$

State variables, output and input parameters

$$z = \begin{pmatrix} u \\ \Phi \\ \dot{u} \\ \dot{\Phi} \end{pmatrix}$$

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General state space model with a limited number of input and output signals

$$\dot{z} = A \cdot z + B \cdot f \qquad y = L \cdot z + U \cdot f$$

State variables, output and input parameters

$$\cancel{z = \begin{pmatrix} u \\ \Phi \\ \dot{u} \\ \dot{\Phi} \end{pmatrix}} \qquad z = \begin{pmatrix} u \\ \dot{u} \end{pmatrix} \qquad y = \begin{pmatrix} u \\ \Phi \end{pmatrix} \qquad f = \begin{pmatrix} F \\ Q \end{pmatrix}$$

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System and input matrix

State equation according to choosen system variables

$$\begin{pmatrix} \dot{u} \\ \ddot{u} \end{pmatrix} = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} \cdot \begin{pmatrix} u \\ \dot{u} \end{pmatrix} + \begin{pmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{pmatrix} \cdot \begin{pmatrix} F \\ Q \end{pmatrix}$$

Equations of motion

$$\begin{aligned} M \cdot \ddot{u} \ + \ D \cdot \dot{u} \ + \ K_{uu} \cdot u + K_{u\Phi} \cdot \Phi \ &= \ F \\ K_{u\Phi}^T \cdot u + K_{\Phi\Phi} \cdot \Phi \ &= \ Q \end{aligned}$$

Elimination of the electrical DOF

$$\Phi = -K_{\Phi\Phi}^{-1} \cdot K_{u\Phi}^T \cdot u + K_{\Phi\Phi}^{-1} \cdot Q$$

$$M \cdot \ddot{u} + D \cdot \dot{u} + \left(K_{uu} - K_{u\Phi} \cdot K_{\Phi\Phi}^{-1} \cdot K_{u\Phi}^T\right) \cdot u = F - K_{u\Phi} \cdot K_{\Phi\Phi}^{-1} \cdot Q$$

Comparision and determination of the matrices of the system equation

$$\begin{pmatrix} \dot{u} \\ \ddot{u} \end{pmatrix} = \begin{pmatrix} 0 & I \\ -M^{-1} \cdot \left(K_{uu} - K_{u\Phi} \cdot K_{\Phi\Phi}^{-1} \cdot K_{u\Phi}^T\right) & -M^{-1} \cdot D \end{pmatrix} \begin{pmatrix} u \\ \dot{u} \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ M^{-1} & -M^{-1} \cdot K_{u\Phi} \cdot K_{\Phi\Phi}^{-1} \end{pmatrix} \begin{pmatrix} F \\ Q \end{pmatrix}$$

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Output and feedthrough (direct transmission) matrix

Output equation of the state space model according to choosen system variables

$$\begin{pmatrix} u \\ \Phi \end{pmatrix} = \begin{pmatrix} L_{11} & L_{12} \\ L_{21} & L_{22} \end{pmatrix} \cdot \begin{pmatrix} u \\ \dot{u} \end{pmatrix} + \begin{pmatrix} U_{11} & U_{12} \\ U_{21} & U_{22} \end{pmatrix} \cdot \begin{pmatrix} F \\ Q \end{pmatrix}$$

Equation for eliminiation of Φ

$$\Phi = -K_{\Phi\Phi}^{-1} \cdot K_{u\Phi}^{T} \cdot u + K_{\Phi\Phi}^{-1} \cdot Q$$

Comparision and determination of the matrices of the output equation

$$\begin{pmatrix} u \\ \Phi \end{pmatrix} = \begin{pmatrix} I & 0 \\ -K_{\Phi\Phi}^{-1} \cdot K_{u\Phi}^{T} & 0 \end{pmatrix} \cdot \begin{pmatrix} u \\ \dot{u} \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & K_{\Phi\Phi}^{-1} \end{pmatrix} \cdot \begin{pmatrix} F \\ Q \end{pmatrix}$$

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- Use of reduced matrices
- State vector z consist of structural master-DOF and according velocities

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Determination of the relevant matrices

- General
  - Model reduction by superelement generation within finite element code
  - Definition of structural master-DOF to represent dynamic behaviour
  - Definition of electrical master-DOF to represent piezo-sensors and -actuators
  - Definition of »dummy« (unity) forces and charges on all mechanical and electrical master-DOF, which build input and/or output signals of the state space model

  $\Rightarrow$ Load vector as list of master-DOF and as input/output filter for state space model
- Step 1 (dynamic)
  - Without active electrical master-DOF
  - All electrical DOF set to zero (short-circuit)

  $\Rightarrow$ Determination of the mass (M), damping (D) and stiffness (K_{uu}) matrices
- Step 2 (static)
  - With active electrical master-DOF
  - All electrical master-DOF without voltage boundary condition (open-circuit)

  $\Rightarrow$ Determination of the full stiffness matrix $\begin{pmatrix} K_{uu} & K_{u\Phi} \\ K_{u\Phi}^T & K_{\Phi\Phi} \end{pmatrix}$

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Modal approach

Modal analysis of an undamped, piezo-mechanical system with eliminated electrical DOF (open-circuit)

$$M \cdot \ddot{u} + K_{eff} \cdot u = 0 \qquad \text{with:} \quad K_{eff} = \left(K_{uu} - K_{u\Phi} \cdot K_{\Phi\Phi}^{-1} \cdot K_{u\Phi}^{T}\right)$$

Diagonal matrix of the eigenvalues

$$\Lambda = \mathrm{diag}(\omega_i^2)$$

Matrix of the eigenvectors of the mechanical DOF normalized with the mass matrix

$$V_u^T \cdot M \cdot V_u = I$$

Matrix of the eigenvectors of the electrical DOF according to substitution (with $Q = 0$)

$$\begin{aligned} \Phi &= -K_{\Phi\Phi}^{-1} \cdot K_{u\Phi}^{T} \cdot u \\ V_\Phi &= -K_{\Phi\Phi}^{-1} \cdot K_{u\Phi}^{T} \cdot V_u \end{aligned}$$

Modal coordinates

$$q = V_u^{-1} \cdot u$$

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Transformation to modal coordinates I

Transformation of the equation of motion

$$\underbrace{V_u^T \cdot M \cdot V_u}_{I} \cdot \underbrace{V_u^{-1} \cdot \ddot{u}}_{\ddot{q}} + \underbrace{V_u^T \cdot K_{eff} \cdot V_u}_{\Lambda} \cdot \underbrace{V_u^{-1} \cdot u}_{q} = V_u^T \cdot F \underbrace{- V_u^T \cdot K_{u\Phi} \cdot K_{\Phi\Phi}^{-1}}_{+\ V_\Phi^T} \cdot Q$$

$$I \cdot \ddot{q} + \Lambda \cdot q = V_u^T \cdot F + V_\Phi^T \cdot Q$$

Transformation of the state variables, output and input parameters

$$z_q = \begin{pmatrix} q \\ \dot{q} \end{pmatrix} \qquad y = \begin{pmatrix} u \\ \Phi \end{pmatrix} \qquad f = \begin{pmatrix} F \\ Q \end{pmatrix}$$

Transformation of the state space equation

$$\begin{pmatrix} \dot{q} \\ \ddot{q} \end{pmatrix} = \begin{pmatrix} 0 & I \\ -\Lambda & 0 \end{pmatrix} \cdot \begin{pmatrix} q \\ \dot{q} \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ V_u^T & V_\Phi^T \end{pmatrix} \cdot \begin{pmatrix} F \\ Q \end{pmatrix}$$

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Transformation to modal coordinates II

Output equation for state space model with physical states

$$\begin{pmatrix} u \\ \Phi \end{pmatrix} = \begin{pmatrix} I & 0 \\ -K_{\Phi\Phi}^{-1} \cdot K_{u\Phi}^{T} & 0 \end{pmatrix} \cdot \begin{pmatrix} u \\ \dot{u} \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & K_{\Phi\Phi}^{-1} \end{pmatrix} \cdot \begin{pmatrix} F \\ Q \end{pmatrix}$$

Matrix of the eigenvectors of electrical DOF according to substitution (with $Q = 0$)

$$\begin{aligned} \Phi &= -K_{\Phi\Phi}^{-1} \cdot K_{u\Phi}^{T} \cdot u \\ V_{\Phi} &= -K_{\Phi\Phi}^{-1} \cdot K_{u\Phi}^{T} \cdot V_{u} \end{aligned}$$

Modal coordinates

$$V_u \cdot q = u$$

Output equation for state space model with modal states

$$\begin{pmatrix} u \\ \Phi \end{pmatrix} = \begin{pmatrix} V_u & 0 \\ V_{\Phi} & 0 \end{pmatrix} \cdot \begin{pmatrix} q \\ \dot{q} \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & K_{\Phi\Phi}^{-1} \end{pmatrix} \cdot \begin{pmatrix} F \\ Q \end{pmatrix}$$

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State space model with modal states

State space equation

$$\begin{pmatrix} \dot{q} \\ \ddot{q} \end{pmatrix} = \begin{pmatrix} 0 & I \\ -\Lambda & -D_q \end{pmatrix} \cdot \begin{pmatrix} q \\ \dot{q} \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ V_u^T & V_\Phi^T \end{pmatrix} \cdot \begin{pmatrix} F \\ Q \end{pmatrix}$$

Output equation

$$\begin{pmatrix} u \\ \Phi \end{pmatrix} = \begin{pmatrix} V_u & 0 \\ V_\Phi & 0 \end{pmatrix} \cdot \begin{pmatrix} q \\ \dot{q} \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & K_{\Phi\Phi}^{-1} \end{pmatrix} \cdot \begin{pmatrix} F \\ Q \end{pmatrix}$$

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State space model with modal states

State space equation

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Output equation

$$\begin{pmatrix} u \\ \Phi \end{pmatrix} = \begin{pmatrix} V_u & 0 \\ V_\Phi & 0 \end{pmatrix} \cdot \begin{pmatrix} q \\ \dot{q} \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & K_{\Phi\Phi}^{-1} \end{pmatrix} \cdot \begin{pmatrix} F \\ Q \end{pmatrix}$$

- Direct results of a numerical modal analysis

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State space model with modal states

State space equation

$$\begin{pmatrix} \dot{q} \\ \ddot{q} \end{pmatrix} = \begin{pmatrix} 0 & I \\ -\Lambda & -D_q \end{pmatrix} \cdot \begin{pmatrix} q \\ \dot{q} \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ V_u^T & V_\Phi^T \end{pmatrix} \cdot \begin{pmatrix} F \\ Q \end{pmatrix}$$

Output equation

$$\begin{pmatrix} u \\ \Phi \end{pmatrix} = \begin{pmatrix} V_u & 0 \\ V_\Phi & 0 \end{pmatrix} \cdot \begin{pmatrix} q \\ \dot{q} \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & K_{\Phi\Phi}^{-1} \end{pmatrix} \cdot \begin{pmatrix} F \\ Q \end{pmatrix}$$

- Direct results of a numerical modal analysis
- Consideration of modal damping, constant damping and/or Rayleigh damping

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State space model with modal states

State space equation

$$\begin{pmatrix} \dot{q} \\ \ddot{q} \end{pmatrix} = \begin{pmatrix} 0 & I \\ -\Lambda & -D_q \end{pmatrix} \cdot \begin{pmatrix} q \\ \dot{q} \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ V_u^T & V_\Phi^T \end{pmatrix} \cdot \begin{pmatrix} F \\ Q \end{pmatrix}$$

Output equation

$$\begin{pmatrix} u \\ \Phi \end{pmatrix} = \begin{pmatrix} V_u & 0 \\ V_\Phi & 0 \end{pmatrix} \cdot \begin{pmatrix} q \\ \dot{q} \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & K_{\Phi\Phi}^{-1} \end{pmatrix} \cdot \begin{pmatrix} F \\ Q \end{pmatrix}$$

- Direct results of a numerical modal analysis
- Consideration of modal damping, constant damping and/or Rayleigh damping
- Static stiffness of the electrical subsystem (capacitance matrix)

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Determination of the relevant data

- General
  - No explicit model reduction necessary
  - Definition of a component of nodes, the DOF of which build input and/or output signals of the state space model
- Step 1
  - Modal analysis of the open-circuit system

  ⇒ Eigenfrequencies and eigenvectors of the input/output DOF
- Step 2
  - All mechanical DOF set to zero
  - Static analysis of the electrical subsystem with loadsteps of unity/zero combinations for all electrical input/output DOF

  ⇒ Reaction charges of the loadsteps represent static electric stiffness ($K_{\Phi\Phi}$)

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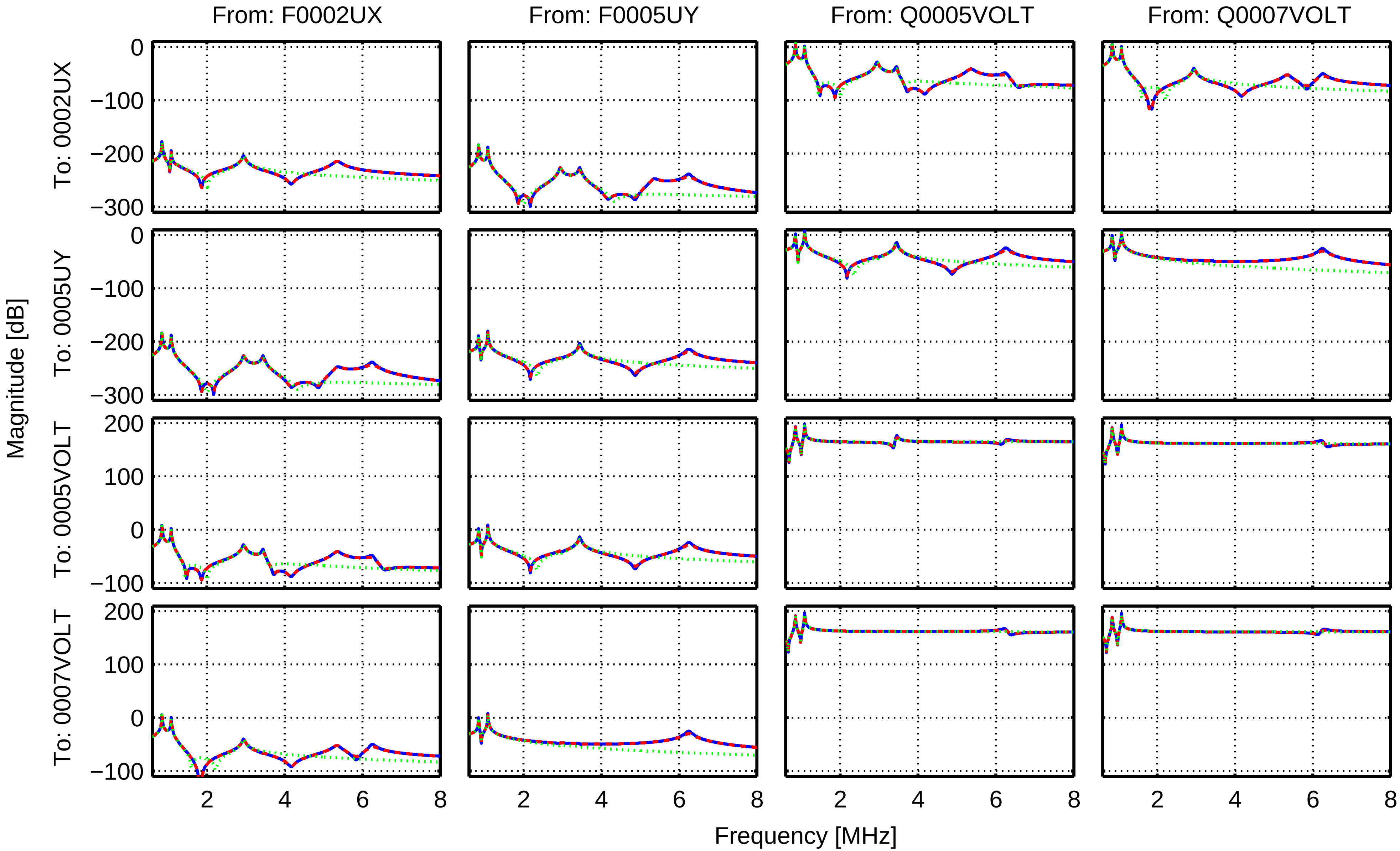
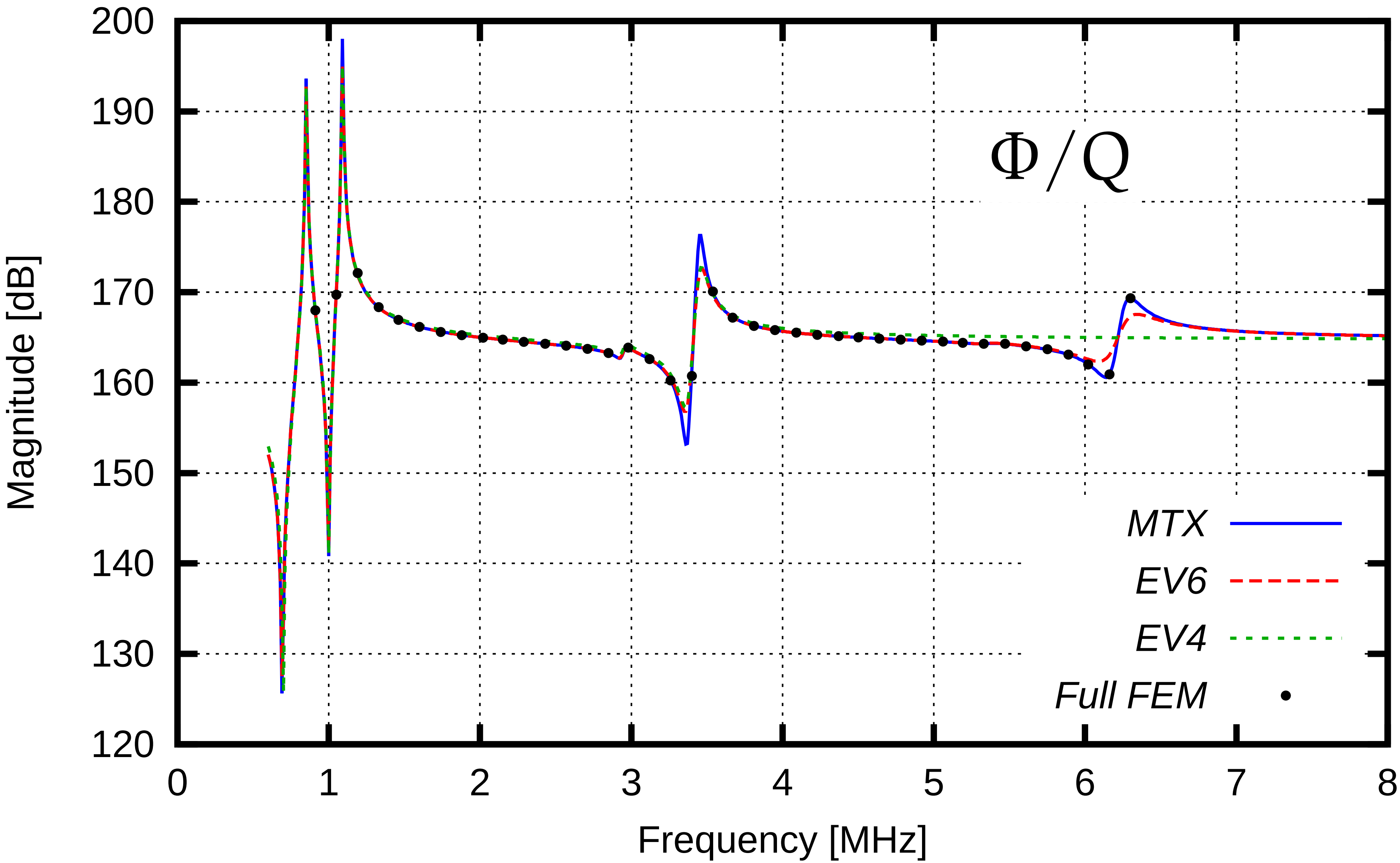
Frequency response functions of the state space models

- 2D-FE-model with 6 mechanical DOF and 3 electrical DOF
- S1: mass, damping and stiffness matrix
- S2: 6 eigenvalues
- S3: 4 eigenvalues



Comparision of the state space models with FE-results



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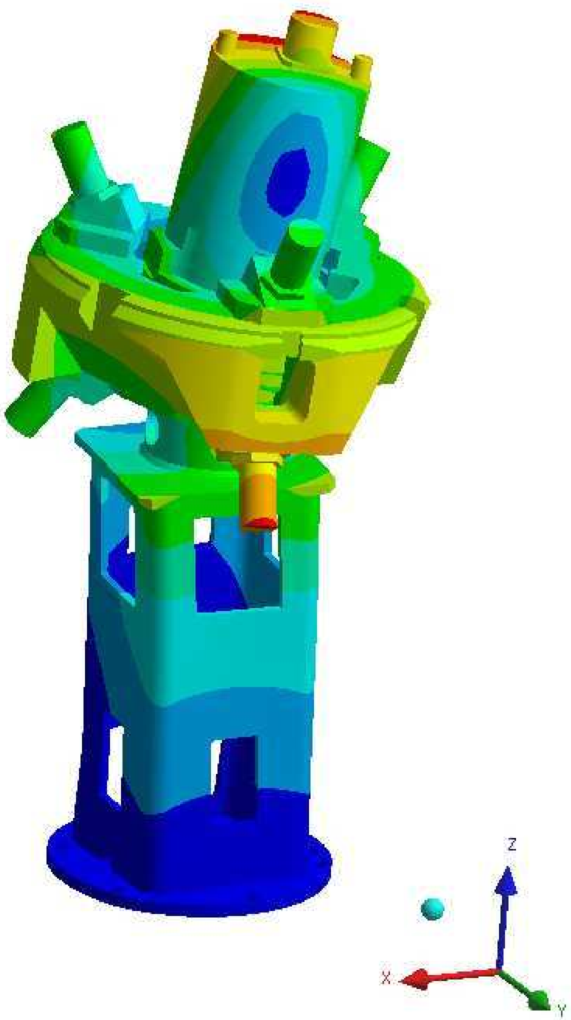
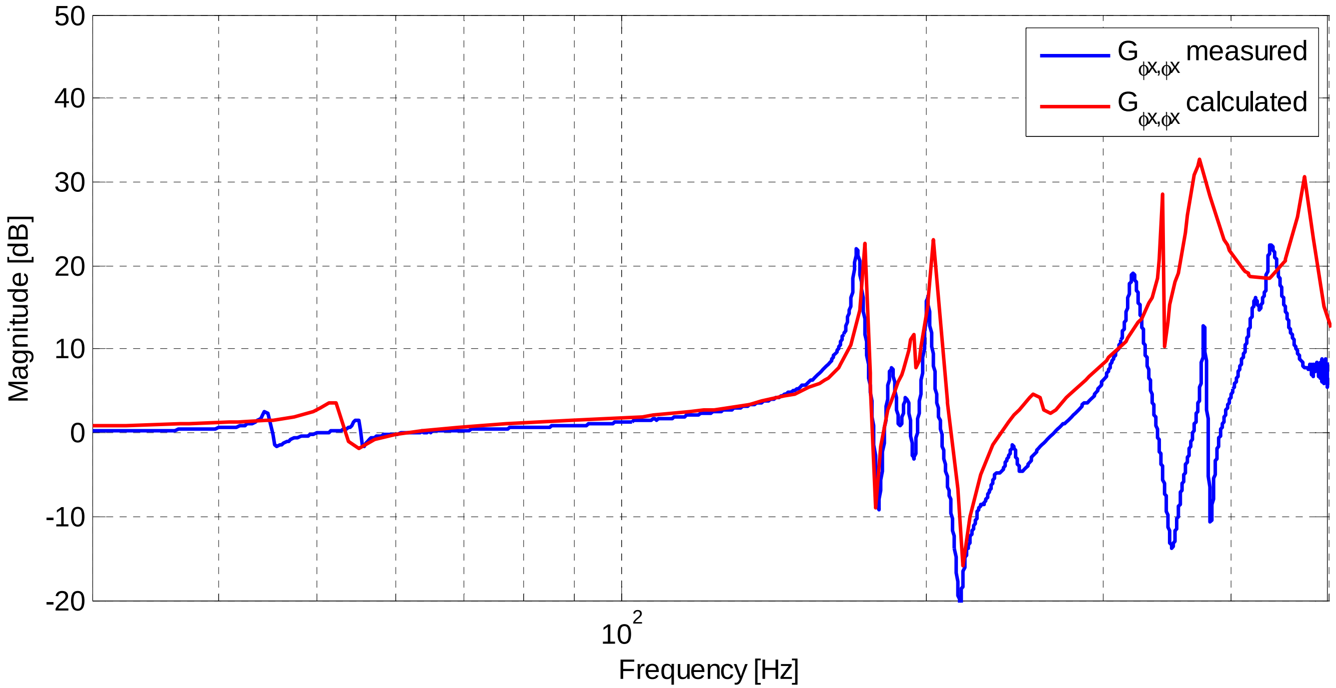
Experimental application





- Transfer function for tilting about the x-axis
- Electrical input signal to tilting of the tool center point

- [Neugebauer2010]: to be published in CIRP Ann.

Conclusion

- Two methods to generate MIMO state space models of piezo-mechanical systems based on a finite-element-discretization
- Correct representation of the electrical impedance
- Method 1: based on superelement matrices
  - Problem: selection of master-DOF
  - Consideration of all types of mechanical damping (also discrete damper)
- Method 2: based on modal analysis and static electrical analysis
  - High efficient
  - Model reduction based on modal truncation
  - Consideration of constant, modal and Rayleigh damping

Outlook

- Usage of the state space model for design of shunt damping circuits
- Different kind for model reduction

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  »Production Technologies for light metal and fibre reinforced composite based components with integrated PIEzoceramic Sensors and Actuators«
- Sponsored by German Research Foundation (Deutsche Forschungsgemeinschaft, DFG)

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Thank you

for your attention!