Joint Source and Channel Coding with Multiple Description Transform Codes

HOLGER HUTZELMANN, ROBERT KLINSKI, RUDI KNORR
Fraunhofer Institute for Communication Systems
Hansastr. 32, 80686 Munich
GERMANY
[hutzelmann, klinski, knorr]@esk.fhg.de   http://www.esk.fhg.de

Abstract: - Multiple description (MD) codes have been proposed as a mechanism to provide robustness against loss of data in the context of packet based networks. Therefore correlation is inserted between the descriptions produced by the MD code. In this paper the focus is on the transmission of multiple descriptions over noisy channels. We investigate the usability of the correlation between the descriptions to combat channel impairments. A MD transform coder (MDTC) is combined with an a priori/a posteriori soft-output Viterbi algorithm (APRI-SOVA) which uses the correlation generated by the MDTC for better decoding. We demonstrate that the correlation between the descriptions can be used for improved decoding at the receiver. Hence multiple description codes may also be viewed as joint source channel codes.

Key-Words: - Joint Source-Channel Coding, Multiple Description Coding, APRI-SOVA

1 Introduction

Multiple Description Coding (MDC) is a source coding method, which is used to produce multiple descriptions of a source. In the initial problem formulation two descriptions have to be transmitted over two different channels to three receivers (Fig. 1). The design of the descriptions should be in a way such that if only one channel works, the information is sufficient to guarantee a minimum reconstruction fidelity. However, should both channels work, the information from both channels can be combined to get a better quality of reconstruction. This can be achieved by introducing some correlation between both descriptions. Several theoretical investigations can be found [5], and practical code designs appear in [2], [7]. In the majority of recent work, the focus is on transmitting the descriptions on packet networks.

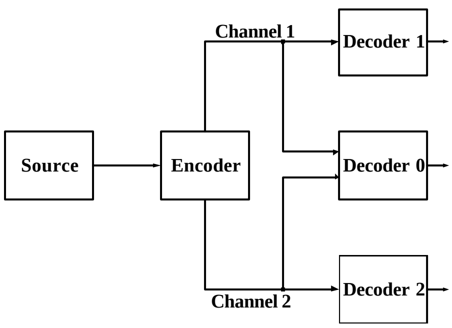

Fig. 1: Basic scenario for multiple description source coding

In [3], Srinivasan demonstrated that the correlation, which is introduced by MDC, can be used effectively on noisy channels. He showed, that the correlation between the descriptions can be used to overcome channel impairments.

In this work we propose a combination of a Multiple Description source coder with an APRI-SOVA [1] and demonstrate that the correlation introduced by the MD source coder can be utilized to improve channel decoding. The rest of the paper is organized as follows. In section 2, a short overview of the APRI-SOVA is given. In section 3 a survey on MDC and the pratical MD code used in the proposed system is given. The proposed approach for combining MDTC and APRI-SOVA is specified in section 4 In section 5 selected simulation are presented illustrating the results of the proposed approach. Concluding remarks are given in section 6.

2 The APRI-SOVA

We consider the transmission system in Fig. 2 consisting of a source encoder, channel coder, AWGN channel, channel decoder and source decoder. A source decoder delivers a source bitstream to the channel coder which encodes the bitstream to protect it from transmission errors. After transmission over a AWGN channel, the channel decoder tries to correct possible transmission errors. The channel decoder used in our system is a Soft-Output Viterbi Algorithm (SOVA), introduced in 1989 by Hagenauer and Hoeher [10]. The SOVA allows to obtain reliability information $L(\hat{u})$ about the decoded information bits $\hat{u}$. A further extension of the SOVA was proposed by Hagenauer [1] in 1995. With a slight modification of the metric of the Viterbi Algorithm, Hagenauer incorporates a priori information $L(u)$ about the probability of the source bits u into the Viterbi Algorithm.

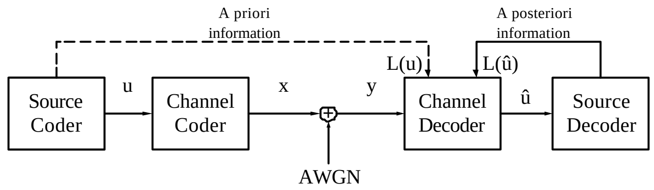

Fig. 2: Model of the transmission system

The metric of the m-th path at time index k of the APRI-SOVA is given by [1]

$$M_k^{(m)} = M_{k-1}^{(m)} + \sum_{n=1}^{N} \left(x_{k,n}^{(m)} \cdot L_{c_{k,n}} \cdot y_{k,n} + u_{k,n}^{(m)} \cdot L(u_k) \right). \quad (1)$$

The value $L_{c_{k,n}} y_{k,n}$ in (1) is the so called soft-output of the channel, where for the AWGN channel $L_{c_{k,n}}$ (the reliability value of the channel) is [1]

$$L_{c_{k,n}} = \frac{4E_b}{N_0}. \quad (2)$$

In [1], it is described how the Viterbi Algorithm will use the a priori information $L(u)$. If the channel is very good $L_{c_{k,n}}$ is larger than $|L(u)|$ and decoding relies on the received channel values. If the channel is very bad, decoding relies on the a priori information $L(u)$ of the source. This circumstance is pictured in Fig. 3.

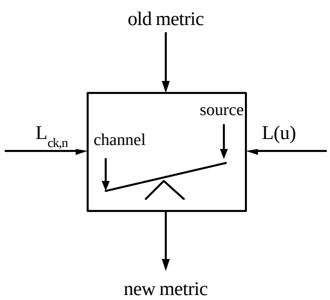

Fig. 3: Weighting property of the APRI-SOVA

This a priori information about the source bits leads to a significantly decreasing bit error rate in the Viterbi Algorithm, especially in a low signal-to-noise environment [1].

We mentioned in section 1, that MDC introduces correlation between the generated descriptions. Therefore it seems natural to ask if the correlation in the source signal can be used by the APRI-SOVA to improve the channel decoding process. Hence, we focus in the following section on the properties of MDC.

3 Multiple Description Coding

3.1 Principle

Multiple Description Coding (MDC) refers to the scenario depicted in Fig. 1. An encoder is given a source sequence to communicate to three receivers over two error-free channels [2].

The encoder generates two distinct yet correlated descriptions and sends both over each channel. The transmission rate over channel i is denoted by $R_i, i = 1,2$. Decoder 0 receives the information which is sent over both channels while the remaining receivers (decoder 1/2) receive only the information over their respective channels. If both channels work, decoder 0 produces a reconstruction sequence with distortion D_0, the central distortion. If one of the channels fail, decoder 1/2 reconstructs the original information with distortion $D_{1/2}$, the side distortion.

The central theoretical problem is to determine the set of achievable values for the quintuple $(R_1, R_2, D_0, D_1, D_2)$, the multiple description rate distortion region. It should be noted, that the MD rate distortion region is completely known only for memoryless Gaussian sources and the squared error distortions measures. For a memoryless Gaussian source with unit variance the MD region is described by [5]

$$\begin{aligned} & D_i \geq 2^{-2R_i} \ ; i = 1,2 \\ & D_0 \geq \frac{2^{-2(R_1+R_2)}}{1-\left(\sqrt{\Pi}-\sqrt{\Delta}\right)^2} \end{aligned} \quad (3)$$

with $\Pi = (1-D_1)(1-D_2)$ and $\Delta = D_1 D_2 - 2^{-2(R_1+R_2)}$.

There have been many contributions [5] consider the theoretical bounds on the performance of MD codes. Among the practical codes which have been proposed, there are two main approaches: Multiple Description Scalar Quantizer (MDSQ) and Multiple Description Transform Coding (MDTC). MDSQ first introduced by Vaishampayan [7] can be seen as the use of a pair of independent scalar quantizers to give two descriptions of a scalar source sample. A different approach to MDC is MDTC [8] where the multiple description character is achieved by a linear transform that introduces correlation between a pair of random variables. If one of the random variables is lost, the correlation between the variables can be used to estimate the lost one. In this work the second approach to produce correlated descriptions is used. Hence we will take a closer look at MDTC.

3.2 Multiple Description Transform Coding

Consider two independent Gaussian random variables X_1 and X_2 with variances $s_1 > s_2$. In the MDC case a transform is used to introduce correlations between the random variables transmitted over different channels, so that if one channel is corrupted the lost random variable can be estimated by the one received. The correlated random variables $[Y_1 \quad Y_2]^T$ are given by

$$\begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} = \mathbf{T} \cdot \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}. \qquad (4)$$

In [8] the transformation used is given by

$$\mathbf{T} = \frac{1}{\sqrt{2}} \cdot \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}. \qquad (5)$$

For single-description source coding the mean square-error distortion per component at R Bits per sample would be given by[1] [2]

$$D_0 = \frac{p \cdot e}{6} s_1 s_2 2^{-2R}. \qquad (6)$$

The average distortion $\overline{D}$ using $[X_1 \quad X_2]^T$ when one channel is lost assuming that each channel is equally likely to fail can be calculated by

$$\overline{D} = \frac{1}{2}(D_1 + D_2) = \frac{1}{4}(s_1 + s_2) + \frac{pe}{12} s_1 s_2 2^{-2R}. \qquad (7)$$

The central distortion achieved by using $[Y_1 \quad Y_2]^T$ instead of $[X_1 \quad X_2]^T$ is

$$D_0' = \frac{pe}{6} \cdot \frac{s_1^2 + s_2^2}{2} \cdot 2^{-2R}. \qquad (8)$$

At side decoder 1 Y_2 has to be estimated from Y_1 by using the optimal MSE estimator. Therefore $\overline{D}'$ is approximately [2]

$$\overline{D}' \approx \frac{s_1^2 s_2^2}{s_1^2 + s_2^2} + \frac{pe}{12} s_1 s_2 2^{-2R}. \qquad (9)$$

Comparing the equations (6) and (8), (7) and (9) one can recognized that the second central distortion is worse than the central distortion without using the transform by a constant factor of

$$g = \frac{s_1^2 + s_2^2}{2 s_1 s_2}. \qquad (10)$$

On the other hand the average side distortion is reduced by a factor of g^2 using the transform. This shows that MDTC improves the side distrotion while it degrades the central distortion.

In [2] a general and detailed analysis of sending a two-tuple over two channels was done. The tranforms considered were given by

$$\mathbf{T} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \text{ with } \det(\mathbf{T}) = 1. \qquad (11)$$

In the following we summarize the results obtained in [2] to show how the correlation or redundancy is controlled by the transform $\mathbf{T}$. The minimum average rate to transmit the random variables X_1 and X_2 is

$$R^* = \frac{1}{4} \log_2 \left(s_1^2 s_2^2 \right) + k_\Delta \qquad (12)$$

where the approximated entropy of a Gaussian random variable with variance s^2 quantized with a bin width Δ is [10]

$$\frac{1}{2} \log_2 \left(s^2 \right) + k_\Delta \text{ where } k_\Delta = \frac{1}{2} \log_2 \left(\frac{2pe}{\Delta} \right). \qquad (13)$$

The random variables $[Y_1 \quad Y_2]^T$ in (4) using the transformation in (11) have variances $a^2 s_1^2 + b^2 s_2^2$ and $c^2 s_1^2 + d^2 s_2^2$. Thus we have rate estimates

$$\begin{aligned} R_1 &= H(Y_1) \approx \frac{1}{2} \log_2 \left(a^2 s_1^2 + b^2 s_2^2 \right) + k_\Delta \\ R_2 &= H(Y_2) \approx \frac{1}{2} \log_2 \left(c^2 s_1^2 + d^2 s_2^2 \right) + k_\Delta \end{aligned}. \qquad (14)$$

The difference between $R = \frac{R_1 + R_2}{2}$ and R^* is called redundancy j in [2]. It is the rate added to improve the side distortions and is

$$\begin{aligned} j &= R - R^* = \ldots \\ \ldots &= \frac{1}{4} \log_2 \left(\frac{\left(a^2 s_1^2 + b^2 s_2^2 \right)\left(c^2 s_1^2 + d^2 s_2^2 \right)}{s_1^2 s_2^2} \right). \end{aligned} \qquad (15)$$

[1] Assuming high-rate entropy-coded uniform quantization.

In [2] different transforms were considered to minimize the average distortion for fixed and nonnegative redundancy j. In the case of balanced rates[2] the elements of $\mathbf{T}$ in (11) are

$$\begin{aligned} a &= \pm\sqrt{\frac{s_2}{2s_1}\left(2^{2j}+\sqrt{2^{4j}-1}\right)} \\ b &= \pm\frac{1}{2a} \\ |a| &= |c|,\ |b| = |d| \end{aligned} \qquad (16)$$

As one can see in (16) the transform $\mathbf{T}$ is defined by j an therefore the correlation introduced by the transform can be controlled by the parameter j. In the later the transform defined by (16) will be used.

Since now it is clear how the correlation between descriptions can be controlled, we consider now, how this redundancy can be incorporate into the APRI-SOVA.

4 Combining MDTC and APRI-SOVA

In order to combine MDTC with the APRI-SOVA, the correlation between the descriptions, which are generated by the transform $\mathbf{T}$, has to be used to estimate the a priori information $L(u)$ from the soft output value $L(\hat{u})$.

Let us assume that the two output random variables $[Y_1 \quad Y_2]^T$ of a MDTC are each coded by a bit vector of length N

$$\begin{aligned} Y_1 &= [y_{1,1}, y_{1,2}, \ldots, y_{1,N}] \\ Y_2 &= [y_{2,1}, y_{2,2}, \ldots, y_{2,N}] \end{aligned} . \qquad (17)$$

The bit vectors are BPSK modulated and then grouped in frames before transmitted over the AWGN channel. Fig. 4 shows the bits $y_{i,j}, i = 1,2; j = 1,\ldots,N$ in such a frame. After transmission the bit sequence is passed to the APRI-SOVA. Form the APRI-SOVA we obtain the soft output values $L(y_{1,j})$ for the bits of vector Y_1 from the APRI-SOVA. The value $L(y_{1,j})$ corresponds to the soft output value $L(\hat{u})$. Since the random variables Y_1 and Y_2 are correlated, we assume that the bits of the corresponding bit vectors are also correlated. Therefore we use the $L(y_{1,j})$ to calculate the new reliability values $L(y_{2,j})$ of Y_2, which are then used as a priori information (corresponding to $L(u)$) in (1) for the APRI-SOVA.

$y_{2,N}$	—	$y_{2,2}$	$y_{2,1}$	$y_{1,N}$	—	$y_{1,2}$	$y_{1,1}$

Fig. 4: Framed transmission of the bit vectors

The problem is, how to obtain $L(y_{2,j})$ from $L(y_{1,j})$. Let $P(y_{1/2,j} = \pm 1)$ be the probability that bit $y_{1/2,j}$ is equal to ± 1 and $P(y_{2,j} | y_{1,j})$ the conditional probabilities that for example the bit $y_{2,j} = 0$ if $y_{1,j} = 0$. Then the $P(y_{2,k} = \pm 1)$ are given by

$$\begin{aligned} P(y_{2,k} = +1) &= \sum_{l=0}^{1} P(y_{2,k} = +1 | y_{1,k} = l) \cdot P(y_{1,k} = l) \\ P(y_{2,k} = -1) &= \sum_{l=0}^{1} P(y_{2,k} = -1 | y_{1,k} = l) \cdot P(y_{1,k} = l) \end{aligned} . \qquad (18)$$

After the soft output have been computed by the APRI-SOVA, the corresponding bit probabilities $P(y_{1k} = \pm 1)$ can be computed by

$$P(y_{1,k} = \pm 1) = \frac{1}{1 + e^{\mp L(y_{1,k})}} . \qquad (19)$$

The conditional probabilities in (18), which are depending on the transform $\mathbf{T}$ of the MDTC, are obtained during a training phase. With (18), the reliability information for $y_{2,k}$ required in (1) is then given by

$$L(y_{2,k}) = \log\left(\frac{P(y_{2,k} = +1)}{P(y_{2,k} = -1)}\right). \qquad (20)$$

The L-value calculated according to (20) is then passed to the APRI-SOVA as a priori information about the source bit.

5 Simulation Results

In this section, some simulation results are presented to illustrate the performance of the proposed approach in section 4. In the simulation setup we considered two zero mean Gaussian sources. The sources have independent components with variances $s_1 = 1$ and

[2] Balanced rates means $R_1 = R_2$.

$s_2 = 0.5$. The generated source samples are coded by a MD transform coder with transform $\mathbf{T}$. The elements of the $\mathbf{T}$ are calculates according to (16) with parameter $j \in \{2,3\}$. After natural binary encoding of the transformed components with $N = 5$, the resulting bitstreams are BPSK modulated and transmitted over the AWGN channel. The AWGN channel is characterized by $\frac{E_b}{N_0}$.

The two cases which are simulated, are compared against a reference system which uses a standard Viterbi Algorithm. In the following figures, the results of the reference system are marked as "no a priori information" in contrast to "a priori information" for the proposed system.

Fig. 5 shows the bit error rate performance of the proposed approach for $j = 3$. One can see that for bad channel conditions the gain is up to 1 dB in E_b/N_0. As E_b/N_0 is increased the gain is decreasing. In reducing the correlation generated by MDTC, the gains through using the redundancy for channel decoding decrease considerably. In Fig. 6 where the redundancy j is equal to 2, the gain is only marginal.

Results for the central distortion D_0 for $j = 3$ and $j = 2$ are presented in Fig. 7 and Fig. 8. To determine D_0 we have used a mean squared-error measure. As one can see in Fig. 7, the gain in terms of distortion is up to 0.5 dB for bad channel conditions and vanishes if E_b/N_0 is increased. The gain in E_b/N_0 is up to 0.5 dB. If the redundancy is decreased ($j = 2$, Fig. 8) the gain in terms of distortion is only small, as already observed by the bit error rate performance in Fig. 6.

6 Conclusions

In this paper we have considered a scenario where multiple descriptions are transmitted over bit error channels. In order to reduce the bit error rate of the channel decoder, we propose a system which combines Multiple Description Transform Codes with the APRI-SOVA. A approach was introduced that utilizes the correlation introduced by MDTC in combination with the soft output of the APRI-SOVA to determine a priori information for the decoding process. Hereby we were able to show, that the correlation can be used to improve channel decoding.

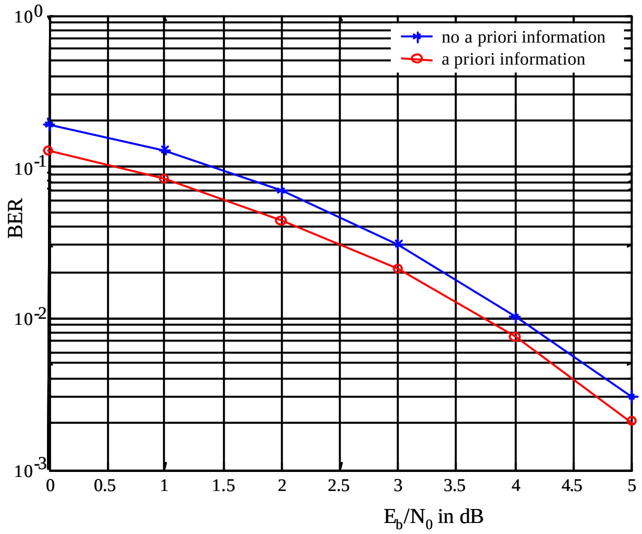

Fig. 5: Bit error rate performance for $j = 3$

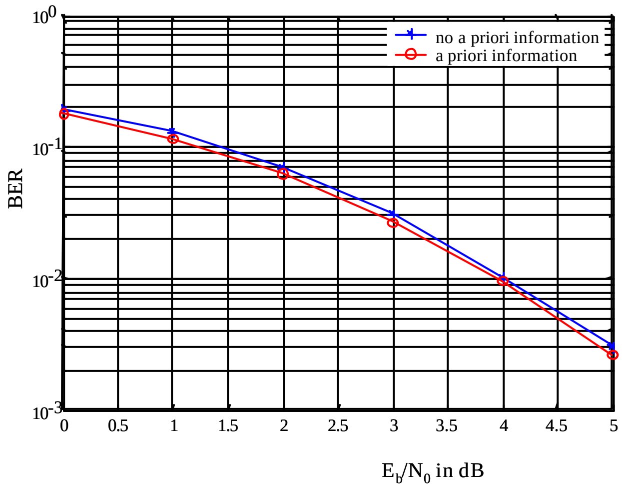

Fig. 6: Bit error rate performance for $j = 2$

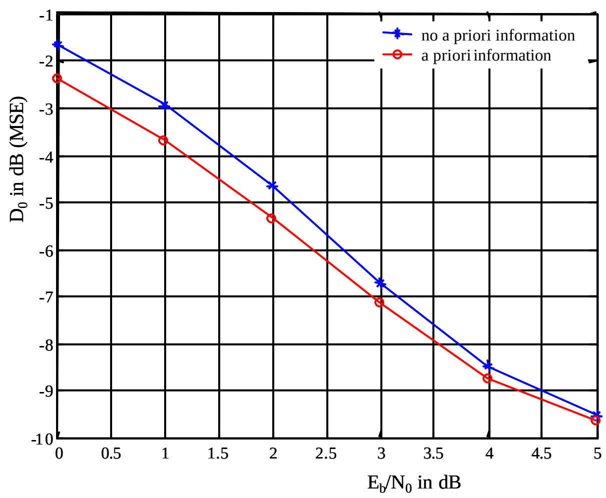

Fig. 7: Central Distortion D_0 in dB (mean square error – MSE) for $j = 3$

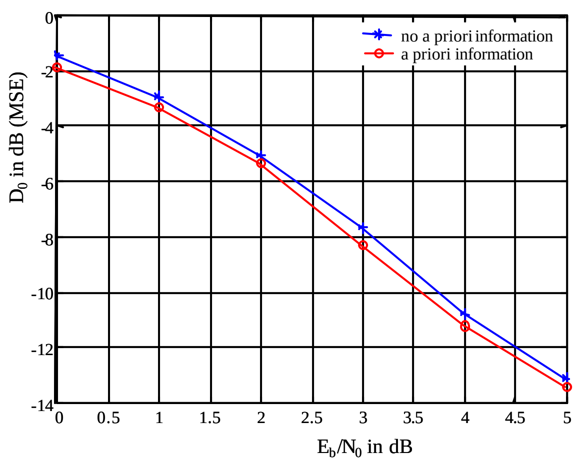

Fig. 8: Central Distortion D_0 in dB (mean square error – MSE) for $j = 2$

References:

[1] J. Hagenauer, „Source-Controlled Channel Decoding“, *IEEE Transactions on Communcations*, vol. 43, no. 9, pp. 2449-2457, September 1995.

[2] V. K. Goyal and J. Kovacevic, „Generalized Multiple Description Coding with Correlating Transforms“, *IEEE Transactions on Information Theory*, vol. 47, no. 6, pp. 2199-2224, September. 2001.

[3] M. Srinivasan, „Iterative Decoding of Multiple Descriptions“, *in Proc. IEEE Data Compression Conference*, pp. 463-472, Snowbird, Utah, March 1999.

[4] S. Emami and S. L. Miller, „DPCM Picture Transmission over noisy Channels with the Aid of a Markov Model“, *IEEE Transactions on Image Processing*, vol. 4, no. 11, November 1995.

[5] L. Ozarow, „On a source-coding problem with two channels and three receivers“, *Bell Systems Technical Journal*, vol. 59, no. 10, pp. 1909-1921, December 1980.

[6] A. A. Gamal and T. M. Cover, „Achievable rates for multiple descriptions“, *IEEE Transactions on Information Theory*, vol. 28, no. 6, pp. 851-857, November 1982.

[7] V.A. Vaishampayan, „Design of Multiple Description Scalar Quantizers“, *IEEE Transactions on Information Theory*, vol. 39, no. 3, pp. 821-834, May 1993.

[8] Y. Wang, M.T: Orchard and A.R. Reibmann, “Multiple description image coding for noisy channels by pairing transform coefficients”, *in Proc. IEEE Workshop on Multimedia Signal Processing*, pp. 419-424, Princeton, New York, June 1997.

[9] M.T. Orchard, Y. Wang, V. Vaishampayan, A.R: Reibman, “Redundancy Rate-Distortion Analysis of Multiple Description Coding Using Pairwise Correlating Transforms”, *in Proc. IEEE Internatinal Conference on Image Processing*, vol. 1, pp. 608-611, Santa Barbara, CA, October 1997.

[10] T.M. Cover, J.A. Thomas, “Elements of Information Theory”, *John Wiley & Sons, Inc*., New York, 1991.

[11] J. Hagenauer, P. Hoeher, „A Viterbi Algortihm with Soft-Decision Outputs and its Applications“, *in Proc. of IEEE Globecom`89*, pp. 47.1.1-47.1.7, November 1989.