# Improvement of Predictive Energy Efficiency Optimization using Long Distance Horizon Estimation

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Abstract—Energy efficiency has become an important topic in trade, transportation and environment protection. Modern electric vehicles usually still have difficulties in reaching similar travel distances as combustion engine powered vehicles. While increasing the range of electric vehicles continues to be an active field of research, it is already possible to increase the energy efficiency by applying a more energy efficient driving behavior. A forward-backward Dynamic Programming model predictive optimization approach is used to generate an energy efficient velocity and gear change trajectory. Due to the finite length of the computation horizon, common Dynamic Programming approaches sometimes have problems of choosing the optimal state at the end of the horizon. To address this problem, a method is presented that makes use of historic accumulated minimum costs to create a separate time-invariant auxiliary horizon that grows during the journey. The auxiliary horizon is used to yield a better long range estimation of the optimal terminal behavior of the optimal trajectory within the regular horizon. While the proposed method can be applied to different types of optimization problems, the focus is on the predictive energy efficiency optimization of electric vehicles.

*Key words*: Dynamic Programming, long distance horizon estimation, reuse historic costs, monetary costs, energy efficiency driving, model predictive optimization, REM 2030 electric vehicle

#### I. INTRODUCTION

Several authors have published solutions in the area of energy efficient driving. The approaches range from complete vehicle control to passive driver assistance systems. Autonomous approaches include the works of [1] [2] [3] [4] [5] [6] [7] and many others. A major class of optimization are formed by discrete planning algorithms [8] like Dynamic Programming (DP) [9]. Examples for predictive energy efficiency optimization include [1] [6] [10] [11].

In this work, the problem of finite computational horizons in DP is addressed. Many predictive optimization methods use some kind of finite computation horizon if the problem refers to a very long planning problem, i.e. the optimal predictive solution is only viable within the finite horizon [1] [2] [3] [6] [10] [11]. Generally, this leads to the question how the optimal solution should behave towards the end of the horizon if the optimal end state is unknown and only a fraction of the entire problem has been solved. Regarding energy efficient driving, an optimal end state at the end of the horizon is often preselected according to some criteria [1] [10] [11]. For example one might choose the speed limit or the optimal stationary cruise velocity as the end state. If the original optimization problem does not dictate the end state at the end of the horizon in advance and if the problem in its entirety extends well beyond the currently defined finite horizon, it is possible that the chosen end state is far away from the true optimal trajectory. From a theoretic point of view, an infinite horizon using continuous problem formulation is often discussed in optimal control problems [9]. An infinite horizon or at least a horizon that leads to the true final ending of the journey may sometimes be infeasible in practice if the driven route is very long. Nevertheless, an approximation of the theoretic problem formulation is viable. In one of our previous works concerning stage-wise Dynamic Programming, we presented a strategy to reuse historic minimum accumulated costs to reduce computational complexity [12]. This idea is reformulated to compute a separate long range time-invariant auxiliary horizon that grows further into the distance during the journey and guides the much shorter regular horizon that contains the optimal trajectory of the vehicle. Another topic discussed in this work, is the definition of monetary costs and the comparability of different types of costs.

The optimization is used in the context of model predictive optimization, velocity trajectory optimization and energy efficient driving of electric vehicles. The REM 2030 electric vehicle power-train [13] provides the model for evaluation purposes. Simulation results based on artificial and real world data are presented to give a preliminary assessment of the proposed methods, while more detailed evaluations have to be reserved for future work as some of our advanced evaluation tools are still under development.

## II. MODEL PREDICTIVE OPTIMIZATION

The goal of energy efficiency optimization of a vehicle's movement trajectory, as defined in this work, is to find a velocity and gear change trajectory that maximizes energy efficiency for a given route. The precise meaning of energy efficiency is described by the cost functional (see section II-C). In this work, forward-backward Dynamic Programming (DP) is used to perform the optimization.

#### A. REM 2030 electric vehicle

The power-train of the REM 2030 electric vehicle has been developed in the project REM 2030 [13]. The maximum speed is capped at 135kph. Some of its prominent features are a small size, low weight synchronous electric engine with a novel cooling system that cools the engine directly at the coils, a two gear transmission, a holistic thermal management

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system that can draw thermal energy from the surrounding and a fuel cell as range extender. The formulation of the energy consumption model in this work is inspired by [10] [14] [15] and defined in an inverse manner. This means the model output is computed from the vehicle's acceleration. Propagation through the model ultimately leads to an energy consumption rate in the batteries. Several characteristic maps are used within the model including the engine's efficiency map. Inertia is evaluated for the engine and the transmission. The resistance forces include air resistance, rolling resistance, slope resistance and acceleration resistance. The battery charge and voltage may vary depending on discharging and charging processes during vehicle operation. The vehicle model is non-linear and no optimization related model simplification is needed. We wish to stress out that the underlying optimization method proposed here, is not confined to a specific vehicle, but can be used for all types of vehicles. Nevertheless, the REM 2030 electric vehicle will serve as an example.

## B. Hamilton-Jacobi-Bellman and Dynamic Programming

The Hamilton-Jacobi-Bellman equation (HJB) is a central part in optimization and control theory. It is both a necessary and sufficient condition for the existence of a global optimal continuous solution. The HJB equation can be formulated as a minimization problem, which requires the continuous differentiability with respect to time t and the state components of the state trajectory function x(t). The minimization commonly refers to the control trajectory function u(t). J(x(t), u(t)) is the accumulated cost functional which is the integral of the transition cost function j(x(t), u(t)). The dynamic system is described by  $\dot{x}(t) = f(x(t), u(t))$ .  $x^*(t)$ ,  $u^*(t)$  and  $J^*(x(t), u(t))$  constitute the optimal solution of the HJB equation:

$$0 = \min_{u(t) \in U} \left( j(x(t), u(t)) + \dot{J}(x(t), u(t)) + \nabla J(x(t), u(t)) f(x(t), u(t)) \right)$$
(1)

The HJB equation is rarely used in practice for highly non-convex, non-linear problems as retrieving the analytical solution is often infeasible. Alternatively, the corresponding discrete solution can be obtained through DP. With ever increasing discretization subtlety, the DP solution converges towards the corresponding HJB solution if the original continuous problem has been formulated as HJB. For further details and background information, the reader may refer to [9]. The continuous cost functional, used in this work, is formulated according to HJB definition, i.e. continuously differentiable with respect to t and the state components x(t). Discretization leads to the discrete time-stamp  $t_i$  with the period  $\Delta t$ .

As discussed later in section II-E, for our optimization approach it is actually better to conduct the discrete optimization with respect to equidistant discrete positions  $s_i$  instead of  $t_i$ . Thus, there is a transformation from discrete time to discrete position and vice versa using the discrete velocity  $v(t_i)$  and  $v(s_i)$ . The discrete optimization yields a chain of optimal decisions that refer to equidistant discrete positions. In order to obtain the solution with respect to time, the inverse transformation is used. Note that the discrete time-stamps after the inverse transformation are not necessarily equidistant, but the time dependent solution itself is valid and increases in precision with increase in discretization subtlety. In this work, the focus is on the computation of a discrete optimal trajectory  $x^*(s_i)$  or rather  $x^*(t_i)$ . The continuous HJB solution  $x^*(t)$  can then be approximated, e.g. through interpolation.

# C. Monetary cost functional

Many studies conducted in the area of energy efficiency optimization use a weighted trade-off between low energy consumption and short travel duration [1] [2] [3] [11]. Weighting parameters often have to be manually adjusted to reach a desired outcome. Furthermore, it can be difficult to decide if there is a net improvement, e.g. if energy consumption is reduced but travel duration increases. In order to make different types of cost terms directly comparable, we propose to convert all cost terms into monetary form. This also means that the HJB cost functional becomes problem specific and is directly derived from financial aspects of an individual or the business model of a company. There is no universal definition of costs, instead the specific problem at hand dictates the definition of the cost functional. Additionally, the cost terms are made HJB compliant, i.e. the initial formulation is in continuous form and the cost terms are continuously differentiable with respect to time t and the state components of x(t). Note that the cost terms presented here, primarily serve as an example for the definition of monetary costs. In general, there is a vast amount of different problem specific formulations as business models and economic problems can be greatly differ from each other. According to our cost definition, costs are positive while gains are negative.

1) Energy: Energy consumption e(x(t), u(t)) can be converted to monetary cost through the price for electricity  $\lambda_e$ . For example, the average electricity cost at electric vehicle charging stations can be used. The energy consumption cost term is therefore

$$j_e(x(t), u(t)) = \lambda_e e(x(t), u(t)) \tag{2}$$

The cost term incorporates both energy consumption and energy generation through recuperation, in which case it receives a negative value.

2) Travel duration and driven distance: Many people are confronted with the every-day task of driving to and from work. The problem specific optimization in this case is computing the minimal cost of the trade-off between  $j_e(x(t), u(t))$  and the cost of time consumption  $j_t(x(t), u(t))$ . Time consumption is the time spent on the journey, that could have been spent at work which results in a loss of revenue. It is assumed that the journey must be made. Thus, travel duration t is not solely used in the

cost term, but rather the difference to the theoretical minimal travel time duration  $t_{min}$  derived from the speed-limits along the route.  $\lambda_t$  is the salary of an employee or the revenue of a company.

$$j_t(x(t), u(t)) = \lambda_t(t - t_{min}) \tag{3}$$

Similarly, costs can also be defined for transport companies or taxi companies, which are paid depending on driven distance and possibly also working hours. On the one hand, the driver wants to quickly finish the delivery in order to make the next delivery possible. On the other hand, driving more slowly, increases the time dependent income. In these cases the theoretical maximal driven distance  $s_{max}(t)$  given a certain travel duration and the maximal travel duration  $t_{max}$ based on the lowest tolerable velocity are of importance.  $\lambda_s$  and  $\lambda_t$  convert the driven distance and time duration into monetary form depending on the business model of the transport company.

$$j_{ts}(x(t), u(t)) = \lambda_s(s_{max}(t) - s(t)) + \lambda_t(t_{max} - t) \quad (4)$$

3) Cost of vehicle and attrition: There are different forms of attrition in a vehicle. First of all, there is the overall value decrease if the vehicle is used to drive a certain distance. A new vehicle has a certain purchasing price C at the beginning and an estimated life expectancy regarding the maximum distance it can be driven before it is scrapped.  $\lambda_C$  is the average value decrease in relation to a driven distance excluding wear on brakes and transmission.

$$j_C(x(t), u(t)) = \lambda_C s(t) \tag{5}$$

Active braking causes attrition to the brake pads depending on the strength of the braking procedure  $f_B(x(t), u(t))$ . Gear change causes attrition to the transmission. Different to brake pads, the transmission is not designed to be replaced multiple times. The attrition  $f_G(x(t), u(t))$  is distributed among numerous parts and does not target a specific element. The average replacement cost in relation to brake attrition and transmission attrition are  $\lambda_B$  and  $\lambda_G$ . If manufacturer specific models are unavailable, person specific statistical frequency of active braking and gear change behavior in combination with average service life can be used as an approximation. In this work, statistical approximation is used.

$$j_B(x(t), u(t)) = \lambda_B f_B(x(t), u(t)) \tag{6}$$

$$j_G(x(t), u(t)) = \lambda_G f_G(x(t), u(t)) \tag{7}$$

4) Safety and lane change: In one of our previous works [16], we have presented preliminary results regarding energy efficiency optimization using lane changes. Lane change optimization and safety shall not be the focus of this contribution. Thorough evaluations of safety aspects in combination with energy efficiency optimization will be presented in future work.

5) Constraints: Apart from the cost functional, several boundary constraints are defined. Speed limits and physical constraints within the power-train such as acceleration or engine full load are of great importance. Lateral acceleration  $a_{lat}(t)$  is related to journey comfort and safety and is currently also incorporated as a constraint. The maximum tolerable lateral acceleration is based on the mean  $\mu_a$  and standard deviation  $\sigma_a$  of driving comfort presented in the field study [17]. The lateral dynamics is currently simplified as a mass point, based on the centripetal force acting on it, depending on the current curvature radius r(t). Certain constraints may be replaced by cost definitions in future work.

$$a_{lat}(t) = \frac{v^2(t)}{r(t)} \tag{8}$$

$$-\mu_a - \sigma_a < a_{lat}(t) < \mu_a + \sigma_a \tag{9}$$

The previously presented cost terms are all HJB compliant, i.e. continuously differentiable with respect to t and x(t). They all describe continuously physical processes subsequently converted to monetary cost. Neither t, u(t), x(t) or anything used in the cost terms can instantly change. Even braking and gear changes need some form of continuous transition, no matter how brief they may be.

#### D. State graph structure

In order to perform the discrete DP optimization, a multistage state graph is defined (Fig. 1). As mentioned in section II-B, equidistant discrete positions instead of discrete timestamps are used in the actual optimization procedure. Within the state graph, the vehicle state  $x(s_i)$  is composed of travel duration  $t(s_i)$ , velocity  $v(s_i)$ , gear level  $G(s_i)$ , brake status  $B(s_i)$  and battery charge  $Q(s_i)$ .

$$x(s_i) = \left(t(s_i), v(s_i), G(s_i), B(s_i), Q(s_i)\right)^T$$
(10)

The state components all refer to a specific position or driven distance  $s_i$ , which also serves as a decision stage within the graph. The stages  $s_i$  within the computation horizon S are erected with a constant frequency, e.g. every 10m. For sake of simplicity, stages and stage positions share the same denotation  $s_i$  in the following sections. Each stage  $s_i$  has a state set  $S_i$  of states  $x_a(s_i)$ . The graph has an open end, i.e. the last stage has as many states as the intermediate stages. The range of possible velocities for a stage from  $s_1$  to  $s_N$  is constrained by the corresponding minimum and maximum velocity limits as well as acceleration and braking constraints of the vehicle. The computation horizon moves with the vehicle. During the forward pass, stagewise forward-backward DP solves the cost minimization problem by evaluating all transition costs from one stage to the previous one. Beginning from the second stage onward, states start working with minimum accumulated costs of the states from the previous stage. The accumulated minimum cost  $J^*(x_b(s_i))$  of a state  $x_b(s_i)$  at stage  $s_i$  is the minimum sum of all possible transition costs  $j(x_a(s_{i-1}), x_b(s_i))$  from predecessor states  $x_a(s_{i-1})$  to  $x_b(s_i)$  and the accumulated minimum cost  $J^*(x_a(s_{i-1}))$  of the respective state  $x_a(s_{i-1})$ . After optimization, every state with the exception of the start state has one unique optimal predecessor state unless constraint violations forbid it. A state may have several or no successors.

$$J^{*}(x_{b}(s_{i})) = \min_{x_{a}(s_{i-1}) \in S_{i-1}} \left( j(x_{a}(s_{i-1}), x_{b}(s_{i})) + J^{*}(x_{a}(s_{i-1})) \right) \quad (11)$$

Finally, let  $x^*(s_N)$  be the optimal end state with the smallest accumulated minimum costs of all end states  $x_a(s_N)$ .

$$x^{*}(s_{N}) = \arg \min_{x_{a}(s_{N}) \in S_{N}} J^{*}(x_{a}(s_{N}))$$
(12)

During the backward pass, the optimal state trajectory is constructed by following the identified chain of optimal states from the last stage to the start state. For further details and background information regarding Dynamic Programming, the reader may refer to [9].



Fig. 1. Graph structure used for forward-backward Dynamic Programming with computation horizon S and decision stages  $s_i$ . The end stage  $s_N$  is open, i.e. it has the same number of states as intermediate stages.

# E. Long range horizon approximation and reuse of historic minimum accumulated costs

As mentioned in the introduction section I, the optimal end state of the DP optimization is difficult to define if a finite computation horizon is used. In one of our previous works, we have presented an adapted forward-backward Dynamic Programming method, which incorporates historic accumulated minimum costs of previous optimizations [12]. A graphic description is given in Fig. 2. Whenever the horizon moves forward, it adds a new decision stage  $s_{N+1}$ to its end and removes its first stage  $s_i$  at the beginning. Only the transitions to the new end stage are evaluated. All other minimum costs on intermediate stages are retrieved from memory. Another difference is that the optimization now refers to the original state, i.e. the vehicle state when the system just started the very first optimization. As the computation horizon moves on, an optimal trajectory that refers to the new start state of the vehicle  $x_1(s_{i+1})$  (blue), is actually needed. But as long as the new optimal trajectory (red) traverses through the new start state, the new optimal trajectory is also optimal with respect to the new start state due to the principle of optimality [9]. If the optimization problem excessively changes during the next optimization, the optimal trajectory may no longer traverse through the new start state. In this case a possibly suboptimal correction of the optimal trajectory (green) is necessary. For scenarios with little traffic and mostly time-invariant properties, corrections are usually unnecessary. But scenarios with dense traffic and erratic events could lead to many unforeseeable corrections.



Fig. 2. Reuse of historic costs (best viewed in color): Decision stages  $s_i$  which refer to the past are discarded. A new decision stage  $s_{N+1}$  is added to the end of the computation horizon. Only the transitions from  $s_N$  to  $s_{N+1}$  are computed. Previously computed accumulated minimum costs are reused. The red trajectory based on historic costs (red) is still referring to the already discarded original state and may not incorporate the current vehicle state  $x_1(s_{i+1})$  in erratic scenarios, in which case a correction (green dashed) is necessary. The corrected trajectory may not be completely identical to the true optimal trajectory (blue) referring to  $x_1(s_{i+1})$ . Simplified schematic boxes referring to this procedure are shown at the bottom.

The method of reusing minimal costs will now be reformulated to compute a long range horizon estimation for the end of the regular optimization horizon S. Instead of applying the idea to the entire optimization process, the DP optimization within a relatively short regular optimization horizon S is always completely executed and incorporates all detectable dynamic obstacles, i.e. other vehicles and traffic lights. This ascertains that the optimal solution within S always refers to the current vehicle state and all state transitions are completely evaluated. Note that the evaluation of dynamic obstacles is not presented in this work, but will be addressed in future work. At the same time, a separate long range time-invariant auxiliary horizon  $\tilde{S}$  is constructed that makes use of historic accumulated minimum costs as described earlier. It uses states  $\tilde{x}(s_i)$  with reduced state dimension, but otherwise has the same discretization degree. Within S all speed limits, topography, curvatures and stationary obstacles are considered, except for dynamic obstacles. A graphic description is given in Fig. 3.

$$\tilde{x}(s_i) = \left(v(s_i), G(s_i), B(s_i), Q(s_i)\right)^T$$
(13)

Before the vehicle's journey is initiated,  $\tilde{S}$  is precomputed based on the chosen route. The length of  $\tilde{S}$  is ideally significantly longer than S and depends on how much time the user is willing to wait before the system is fully operational. During the journey, the optimization is conducted for both Sand  $\tilde{S}$  separately. In  $\tilde{S}$ , only the transitions to the additional stages are evaluated. Any previous costs are left unchanged. At least two additional decision stages must be added to the end of  $\tilde{S}$  with each optimization update to make  $\tilde{S}$  grow in length. The underlying idea is, if there is sufficient memory, the long range cost assessment will eventually reach the end of the route or rather the journey's final destination, which might be hundreds or thousands of kilometers away. As only a few (e.g. two) additional stages with reduced state dimension have to evaluated, the additional computation burden is relatively low, while the cost assessment using Sstretches over a very long distance that becomes even longer with every optimization update. When the optimal trajectory and the optimal next state of the vehicle is computed, S and S have to be merged at the last stage  $s_M$  of S. Based on the position of  $s_M$ , one or several stages at the beginning of S are discarded if their positions are not ahead of  $s_M$ . All state transitions between  $s_M$  and the first remaining stage  $\tilde{s}_i$  of S are evaluated. For the construction of the optimal trajectory in S, the accumulated minimum costs  $J(\tilde{x}_c(\tilde{s}_N))$  at the last stage  $\tilde{s}_N$  of  $\tilde{S}$  are temporarily corrected to  $\tilde{J}(\tilde{x}_c(\tilde{s}_N))$  based on the newly evaluated state transitions between S and  $\tilde{S}$ :

# $\tilde{\tilde{J}}(\tilde{x}_{c}(\tilde{s}_{N})) = \tilde{J}(\tilde{x}_{c}(\tilde{s}_{N})) - \tilde{J}(\tilde{x}_{b}(\tilde{s}_{i})) + J(x_{a}(s_{M})) + j(x_{a}(s_{M}), \tilde{x}_{b}(\tilde{s}_{i}))$ (14)

After the connection between S and  $\tilde{S}$  has been established, the optimal trajectory is retrieved in the backward pass starting at the optimal end state at  $\tilde{s}_N$ , going back through  $\tilde{S}$  and S until the vehicle's current state is reached. Thus, the horizon S is no longer dependent on carefully chosen end velocities or end costs, but uses a significantly longer approximation horizon  $\tilde{S}$  to choose its end state. As soon as  $\tilde{S}$  reaches the journey's final destination, it is assumed that the final state is a full stop and  $\tilde{S}$  stops growing.

While  $\tilde{S}$  has not yet reached the end of the journey, choosing the optimal end state of  $\tilde{S}$  before the merging process, actually still requires a less sophisticated estimation, because  $\tilde{S}$  is already used to guide S and there is not another auxiliary horizon that can guide  $\tilde{S}$ . To handle the route beyond  $\tilde{S}$ , the remaining cost from the end of  $\tilde{S}$  until the end of the journey is estimated by using the velocity of each end state  $\tilde{v}_c(\tilde{s}_N)$  as the constant cruise speed on the remaining route ahead of  $\tilde{S}$ . For computations beyond  $\tilde{S}$ , it is assumed that the slope will be zero on average without curvatures and speed limits are ignored. As all state transitions between Sand  $\tilde{S}$  are fully evaluated and due to the Markov property of the problem, the entire optimal trajectory from the beginning of S to the end of  $\tilde{S}$  is physically viable.

The cooperation between S and  $\hat{S}$  is one of the main reasons, why the optimization is referring to equidistant positions instead of time-stamps. The optimization within S follows the search space reduction strategy presented in one of our previous works [16]. The state dimension is not fully expanded during the search. Some state components are calculated along different trajectories. If equidistant timestamps were used as reference, position would be calculated through accumulation. In this case, the states of a decision stage would have the same time-stamp, but different positions. This would make the transition from S to  $\tilde{S}$  more difficult to compute, because the long range horizon  $\tilde{S}$  uses a position dependent evaluation of the problem. While absolute time is irrelevant, position related influences (e.g. slope) are relevant. Transitions among a possibly large number of stages would have to be evaluated instead of just two. The proposed strategy only works for forward-backward DP and not backward-forward DP as the accumulated costs in backward-forward DP always refer to an often unknown end state.



Fig. 3. Long range auxiliary horizon (best viewed in color): The short range regular horizon S and the long range auxiliary horizon  $\tilde{S}$  are stored separately. While S is always completely reevaluated,  $\tilde{S}$  reuses historic accumulated minimum costs and grows in length by adding two or more new decision stages (green) to the end of the horizon. The optimization at the end of S is guided by  $\tilde{S}$ . Depending on the progression of S, a number of decision stages in  $\tilde{S}$  (grey) are removed that are no longer relevant to S. New state transitions between S and the first relevant stage of  $\tilde{S}$  are computed. A new optimal state trajectory (red) from the start state to the end of  $\tilde{S}$  is constructed based on the new state transitions.

# III. RESULTS

In this section, regular DP optimization and DP optimization using auxiliary long range horizon are compared. The results are obtained from simulation, but data from the real world (e.g. slopes) is included. In the two presented examples, the regular horizon length is 50m with 10 decision stages. The length of S is primarily based on the range of perception sensors, e.g. 50m is the reliable maximum range for certain stereo vision systems in automotive applications [18]. The length of the initial long range auxiliary horizon depends on how much time is available for precomputed evaluation before the journey begins. In this example, S is initially 200m long and grows by two stages with every new update. The global optimal reference is a regular DP optimization result that stretches over the entire journey. The influence of dynamic obstacles, e.g. traffic lights and other vehicles, are not included, but will be discussed in future work. The REM 2030 electric vehicle is used as the vehicle model [16] [13]. It has two gears and the maximum velocity is capped at 135kph. On flat terrain, the optimal constant cruise velocity is 99kph and the optimal gear choice is 2. In the figures, the solid line "DP opt" represents the velocity of the global optimal solution. The red dashed trajectory "DP mix" refers to DP supported by the long range auxiliary horizon. "DP short" is the green dashed trajectory and represents DP without any support. It's terminal costs at the end of the horizon merely evaluate the assumed costs of continuing different end velocities and end gears until the end of the route. It is important to note that to fully understand the chosen decisions, one would have to analyze every single decision made during the optimization process. Numerous different forces and goals can make the optimization results hard to interpret. For example, braking is not necessarily always an inefficient behavior as electric vehicles are able to recuperate energy during the process, albeit not at 100% efficiency. Monetary costs can at least yield a much faster overall assessment. The simulation is implemented in C++ in combination with OpenMP and conducted on a modern PC with Intel Core i7-4790 at 3.6GHz and 16GB RAM. With the chosen settings, an optimization update requires less than 0.2 seconds.

## A. Single hill scenario

The first example uses a simple artificial hill to make the interpretation of the optimization result easier to understand (Fig. 4). There is no speed limit. At the beginning, the vehicle is not moving. The task is to travel over the hill and continue the journey. Initially, both "DP mix" and "DP short" accelerate in a similar fashion. "DP mix" (100kp) is slightly faster than "DP short" (99kph), therefore gaining slightly more kinetic energy in front of the hill. When ascending the hill and then on the plateau before driving down-hill, "DP mix" is slower than "DP short" as the auxiliary horizon has already identified the existence of the descent. Entering the descent with a high velocity can invoke more braking in order to stay below the vehicle's maximum velocity cap. During the descent, "DP mix" coasts to a much higher velocity than "DP short" which gradually transfers into the stationary optimal cruise velocity on the flat terrain behind the hill. "DP short" beyond the horizon is only guided by the rough assumption that the future terrain is flat on average and therefore mostly tries to maintain the optimal stationary cruise velocity for flat terrains. In this example, the result "DP mix" is identical to the global optimal solution "DP opt". The cost savings of "DP mix" compared to "DP short" are: energy 2%, time 0.7%, active braking 83%, gear changes 40%, overall cost reduction 2.3%.

#### B. Real world mountain scenario

This example is also obtained from simulation, but the route truly exists in reality. It is within and outside a village named Bergwald south-east of the city Karlsruhe. The route takes the vehicle through the outer ring in Bergwald, descends down the mountain, goes up the same road back up and ends just in front of the village. The elevation data is obtained via Google Elevation API [19]. The curvature radius is estimated using geographic coordinates that define the route. The speed limits are observed on-site. The results are given in Fig. 5. Compared to the first example, both "DP mix" and "DP short" need to brake more often, primarily due to locations with strong curvature. "DP short" is often faster than "DP mix" as the stationary optimal cruise velocity for flat terrain is often above the true optimal velocity profile. But often a strong acceleration by "DP short" is followed by a rapid deceleration due to unforeseen strong road curvature, whereas "DP mix" is informed early by the auxiliary horizon. In other scenarios with relaxed lateral acceleration constraints, the average velocity is generally higher and active braking less likely.

Once again, the result of "DP mix" is identical to the global optimal solution "DP opt". The cost savings of "DP mix" compared to "DP short" are: energy 14.7%, time -6.2%, active braking 3.4%, gear changes 6%, overall cost reduction 6.7%, i.e. there is a reduction in energy consumption, but at the same time an increase in travel duration. Without monetary costs, such a result is ambivalent. But the monetary cost formulation yields a clear verdict: there is an overall decrease in costs of 6.7%.

#### C. Other scenarios

Numerous other tests with different choice of discretization and horizon length suggest that a very long regular horizon S of several kilometers reduces the benefit of using  $\tilde{S}$ . But this naturally also leads to a longer computation duration. Short regular horizons and significant changes regarding slope, curvature and speed limits increase the necessity of the auxiliary horizon. On a flat terrain without any changes, there is no benefit at all as the optimal constant cruise velocity is the target.



Fig. 4. Optimization trajectories (best viewed in color): Global optimal solution (DP opt), DP with short regular horizon and auxiliary long range horizon support (DP mix), DP only with short regular horizon (DP short). Artificial hill without speed limits. Maximum velocity of REM 2030 electric vehicle is capped at 135kph.

#### **IV. CONCLUSION**

Dynamic Programming with finite horizons has the problem of choosing a suitable end state at the end of the horizon if the predictive optimization problem extends far beyond the computation horizon and if there are multiple possible end states. Furthermore, the finite horizon does not allow the predictive optimization to look beyond it, which can reduce the optimization's effectiveness if problem specific properties evolve over long distances, e.g. mountains. But increasing



Fig. 5. Data referring to a real route in and close to Bergwald village southeast of Karlsruhe, Google Elevation API data, curvature estimated from geographic coordinates, speed limits retrieved through on-site observation. The route leads through the outer ring of Bergwald, descends down the mountain and leads back up to Bergwald.



Fig. 6. Optimization trajectories (best viewed in color): Global optimal solution (DP opt), DP with short regular horizon and auxiliary long range horizon support (DP mix), DP only with short regular horizon (DP short).

the horizon length usually also increases the computation duration of the algorithm. A method has been proposed to enable long range cost approximation far beyond the regular horizon, but does not significantly increase computational complexity. The method does require more memory, but the maximum length of the auxiliary horizon and therefore the memory demand can be flexibly adjusted, depending on the employed computation hardware. Furthermore, monetary costs are proposed that are not reliant on heuristic parameter adjustments and enable an objective, unambiguous and problem specific problem evaluation. The application for the optimization is energy efficient driving of electric vehicles. The REM 2030 electric power-train serves as the model for evaluation purposes. Examples based on simulated and real world data have given a first impression of the effectiveness of the proposed ideas.

Future work will combine all of our research experiences of the past to create a model predictive optimization strategy that enables predictive energy efficient driving. The optimization will include topography, speed limits, curvatures, vehicle specific properties, traffic lights and other traffic participants to create an unified optimization that combines both energy efficiency and safety. Specifically regarding this work, the influence of dynamic obstacles on long range optimization will be investigated.

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