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Vorwort

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In der Reihe »Berichte des Fraunhofer ITWM« soll die Arbeit des Instituts kontinuierlich einer interessierten Öffentlichkeit in Industrie, Wirtschaft und Wissenschaft vorgestellt werden. Durch die enge Verzahnung mit dem Fachbereich Mathematik der Universität Kaiserslautern sowie durch zahlreiche Kooperationen mit internationalen Institutionen und Hochschulen in den Bereichen Ausbildung und Forschung ist ein großes Potenzial für Forschungsberichte vorhanden. In die Berichtreihe sollen sowohl hervorragende Diplom- und Projektarbeiten und Dissertationen als auch Forschungsberichte der Institutsmitarbeiter und Institutsgäste zu aktuellen Fragen der Techno- und Wirtschaftsmathematik aufgenommen werden.

Darüber hinaus bietet die Reihe ein Forum für die Berichterstattung über die zahlreichen Kooperationsprojekte des Instituts mit Partnern aus Industrie und Wirtschaft.

Berichterstattung heißt hier Dokumentation des Transfers aktueller Ergebnisse aus mathematischer Forschungs- und Entwicklungsarbeit in industrielle Anwendungen und Softwareprodukte – und umgekehrt, denn Probleme der Praxis generieren neue interessante mathematische Fragestellungen.

Prof. Dr. Dieter Prätzel-Wolters
Institutsleiter

Kaiserslautern, im Juni 2001

Multi-Period Public Transport Design: A Novel Model and Solution Approaches

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Abstract

In this paper, we are going to propose the first mathematical model for Multi-Period Hub Location Problems (MPHLP). We apply this mixed integer programming model on public transport planning and call it MULTI-PERIOD HUB LOCATION PROBLEM FOR PUBLIC TRANSPORT (MPHLPPT). In fact, HLPPT model proposed earlier by the authors is extended to include more facts and features of the real-life application. In order to solve instances of this problem where existing standard solvers fail, a solution approach based on a greedy neighborhood search is developed. The computational results substantiate the efficiency of our solution approach to solve instances of MPHLPPT.

Key words: Integer programming, hub location, public transport, multi-period planning, heuristics

1 Introduction

Hub-and-spoke networks have shown to receive a lot of attention due to their applications— mainly in transportations, telecommunications. Their specific structure and functionality made them very attractive for many researchers who have been dealing with them. The context of Hub Location Problems

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(HLPs) includes all such configurations in different fields for instance, computer networks, postal-delivery, less-than-truck loading and supply chain management. Applications of HLPs in Public Transport (PT) have received less attention compared to other fields, like telecommunication.
In a HLP network, a flow originated from an origin i and destined to node j is not shipped directly. Rather, flows are conducted to their destinations via some selected intermediate nodes (called *hub nodes*) and edges (called *hub edges*) connecting these hubs. When the hub nodes are selected, the non-hub nodes (called *spoke nodes*) which possess the demands of many destinations will be allocated to them in order to send their flow via hub-level network. The allocation scheme can be single or multiple based on the permission to allocate a spoke node just to a single hub node or more than that, respectively. In the classical HLP models, four main assumptions were always considered:

Ass. a The hub-level network is a complete graph.
Ass. b Using inter-hub connections has a lower price per unit than using spoke connections. That is, it benefits from a discount factor $\alpha, (0 < \alpha < 1)$.
Ass. c Direct connections between the spoke nodes are not allowed.
Ass. d Costs are proportional to the distance (the triangle inequality holds).

For the first time, [28, 29] paved the way for the future study of hub location problems. On the discrete hub location problem, the first work again is due to [30]. He proposed the first mathematical formulation (a quadratic model) for *Single Allocation p-Hub Median Problem* (SApHMP).
There are some reviews devoted to HLP on a discrete network. Among these reviews, we refer the readers to the two latest ones by [8] and [4] and references therein.
For the single allocation variants, we can refer to [30, 32], [6, 7], [35], [15], [13].
For multiple allocation models we refer to [6, 7], [35], [16] and [37].
For the capacitated and other variants we refer the readers to [31], [6], [37], [5], [17], [14], [24], [39, 40], [25] and [11].
Beside a variety of exact solution methods proposed in literature, several heuristics are also developed for the variants of HLPs. Solution procedures based on simulated annealing are reported in [15, 17] and [2]. Genetic algorithms are considered by [3], [38] and [12]. Several variants of tabu search algorithms are also proposed in literature. Instances of these algorithms can be found in [22, 23], [34], [9] and [40]. Greedy Randomized Adaptive Search Procedure (GRASP) is also proposed in [22, 23]. A hybrid of genetic algorithm and tabu search is proposed by Abdinnour-Helm [1] for USAHLP. Another hybrid algorithm in literature is due to [10] for USAHLP. [36] take the advantage of the quadratic model of [30] to map on a Hopfield neural network.
There are also some other proposed heuristics which may not be classified into the well-known categories. Some of these algorithms can be found in [33], [30, 31], [21], [7], [20], [16] and [14]. [19] proposed a very efficient greedy neighborhood search algorithm. Their so called *greedy** algorithm is capable

of finding optimal solutions for all the instances with known optimal solution. To the best of our knowledge, [27] have been the first to propose MIP models for the application of HLPs in urban traffic networks. They proposed two models which are known as Public Transport (PT) and Generalized Public Transport (GPT). In their models some of the classical assumptions of HLPs are relaxed and the models are customized for the public transport planning.
Another model called Hub Location Problems for Public Transport (HLPPT) is also proposed in [18]. By relaxing some classical assumptions and introducing new ones, this model also particularly addresses the public transport application. They showed that their model is superior to [27] and also proposed an efficient Benders decomposition approach to tackle instances of the problem.
The rest of the paper is organized as follows: In the next section after reviewing HLPPT, we are going to propose MPHLPPT as a general multi-period model. This model incorporates and represents more realistic aspects of the real-life application. In Section 3, the computational results upon a set of randomly generated instances for two different variants of the model are reported. In Section 4, a greedy neighborhood search is proposed to solve instances of variants of this problem. In Section 5, a large scale instance of MPHLPPT for each of the variants is tackled by the proposed heuristic giving an impression of the computational time. Finally, in the last section we will conclude our work and will propose some further research directions.

2 Mathematical Model

As mentioned earlier, a so called HLPPT model is proposed for the application of HLPs in public transport planning. Although, in this model they also tried to consider real-life aspects of the application, however all the decisions are addressing a single period planning.
Usually, the construction projects are long-lasting, resource-demanding and finance-demanding projects and during the construction phase some parameters of system change. Very often in reality, decisions are made for a planning horizon with several periods. One can refer to [26] for a multi-period model in supply chain models.
In a Multi-Period Hub Location Problem for Public Transport (MPHLPPT), the public transport network is evolving over the planning horizon. Decisions about how the network should evolve are not made just in an improvident way. That is, the configuration of system in each period might not be optimal for that specific period rather, it drops out the individual period optimality in favor of contribution in the optimality over the planning horizon (overall optimality).

MPHLPPT is essentially adapted from HLPPT which is depicted in the sequel. The variables in HLPPT are defined as follows: $x_{ijkl} = 1$ if the optimal path from i to j traverses the hub edge (k, l) and 0, otherwise. Also, $a_{ijk} = 1$ if the optimal path from i to j traverses the spoke edge (i, k) while i is not hub and 0, otherwise and $b_{ijk} = 1$ if the optimal path from i to j traverses the spoke edge (k, j) while j is not hub and 0, otherwise. In addition, $e_{ij} = 1$ if the optimal path from i to j traverses (i, j) and either i or j is a hub and 0, otherwise. For the hub-level variables, $y_{kl} = 1, k < l$, if the hub edge (k, l) is established, 0 otherwise and $h_k = 1$ if k is used as a hub node, 0 otherwise.
The transportation cost for a given flow with origin i and destination j amounts to the sum of (i) the cost of sending the flow from i to the first hub node, (ii) the cost of traversing one or more hub edges discounted by the factor α $(0 < \alpha < 1)$ and, (iii) the cost of transition on the last spoke edge. The proposed mathematical formulation follows:
(HLPPT)

$$\begin{aligned}
Min \sum_{i}\sum_{j\neq i}\sum_{k}\sum_{l\neq k} \alpha W_{ij} C_{kl} x_{ijkl} + \sum_{i}\sum_{j\neq i}\sum_{k\neq i,j} W_{ij} C_{ik} a_{ijk} + \\
\sum_{i}\sum_{j\neq i}\sum_{k\neq i,j} W_{ij} C_{kj} b_{ijk} + \sum_{i}\sum_{j\neq i} W_{ij} C_{ij} e_{ij} + \\
\sum_{k} F_k h_k + \sum_{k}\sum_{l>k} I_{kl} y_{kl} && (1)
\end{aligned}$$

$$\begin{aligned}
s.t. \ & \sum_{l\neq i} x_{ijil} + \sum_{l\neq i,j} a_{ijl} + e_{ij} = 1, && \forall i, j \neq i, (2)\\
& \sum_{l\neq j} x_{ijlj} + \sum_{l\neq i,j} b_{ijl} + e_{ij} = 1, && \forall i, j \neq i, (3)\\
& \sum_{l\neq k,i} x_{ijkl} + b_{ijk} = \sum_{l\neq k,j} x_{ijlk} + a_{ijk}, && \forall i, j \neq i, k \neq i, j, (4)\\
& y_{kl} \leq h_k, \quad y_{kl} \leq h_l, && \forall k, l > k, (5)\\
& x_{ijkl} + x_{ijlk} \leq y_{kl}, && \forall i, j \neq i, k, l > k, (6)\\
& \sum_{l\neq k} x_{kjkl} \leq h_k, && \forall j, k \neq j, (7)\\
& \sum_{k\neq l} x_{ilkl} \leq h_l, && \forall i, l \neq i, \ (8)\\
& e_{ij} \leq 2 - (h_i + h_j), && \forall i, j \neq i, \ (9)\\
& a_{ijk} \leq 1 - h_i, && \forall i, j \neq i, k \neq i, j, (10)\\
& b_{ijl} \leq 1 - h_j, && \forall i, j \neq i, l \neq i, j, (11)\\
& a_{ijk} + \sum_{l\neq j,k} x_{ijlk} \leq h_k, && \forall i, j \neq i, k \neq i, j, (12)\\
& b_{ijk} + \sum_{l\neq k,i} x_{ijkl} \leq h_k, && \forall i, j \neq i, k \neq i, j, (13)\\
& e_{ij} + 2x_{ijij} + \sum_{l\neq j,i} x_{ijil} + \sum_{l\neq i,j} x_{ijlj} \leq h_i + h_j, && \forall i, j \neq i, (14)
\end{aligned}$$

$$x_{ijkl}, y_{kl}, h_k, a_{ijk}, b_{ijk}, e_{ij} \in \{0, 1\}. \tag{15}$$

In the multi-period model (MPHLPPT), in addition to the existing assumptions for the single period problem (i.e. connectivity of hub-level network and independency from any special cost structure in HLPPT), the following assumptions are additionally considered:

(1) the transport network includes an initial configuration,
(2) at most once, the status of each hub node or hub edge can change:
  (a) if a facility exists in the initial configuration it may become closed afterwards,
  (b) if a facility became close at a period in the planning horizon, it remains closed until the end of planning horizon and,
  (c) if a facility became open at a period over the planing horizon it will not be subjected to removal.
(3) a fixed maintenance cost is incurred for using a hub node or hub edge in each period (that means, an amount of budget is considered periodically for maintenance. The vehicles, roads and rails, stations, buildings and many more items are subjected to inspection, control and renewal) and,
(4) a fixed ceasing (removal) cost is incurred in order to degrade a hub node or hub edge to a spoke one.

Some parameters and variables for MPHLPPT are introduced in the following.

2.1 Parameters

In order to extend the model to a more general case, which assumes setup, maintenance and removal costs for both hub nodes and hub edges in each period, the following parameters are introduced:

- HMC_k^t: The maintenance cost incurred by k-th hub node at the t-th period,
- EMC_e^t: The maintenance cost incurred by e-th hub edge at the t-th period,
- HCC_k^t: The removal cost incurred by k-th hub at the t-th period and,
- ECC_e^t: The hub edge removal cost incurred by e-th hub edge at the t-th period,

where $e \in E = \{(k, l)|k, l = 1, \ldots, n, l > k\}$.

2.2 Variables

In order to reflect the requirements of this new model, the definition of variables should be revised. According to the assumptions of model concerning the existence of an initial configuration, two sets of facilities are imagined. Two index sets for facilities are H and E, keeping track of indices of potential hub nodes and potential hub edges, respectively. Each set is partitioned into two subsets. The subset composed of indices of facilities which can be opened (say *openable*) at any $t \in \mathcal{T}$ over the planning horizon labeled with a superscript ^{o}, like H^o and E^o; and the sets of those which are active in the initial configuration or in the other words can be closed later on (say *closeable*), labeled by a superscript ^{c}, like H^c and E^c. This partitioning implies:

$$E^c \cap E^o = \emptyset, \quad E^c \cup E^o = E, \tag{16}$$
$$H^c \cap H^o = \emptyset, \quad H^c \cup H^o = H. \tag{17}$$

In addition, we revise the definition of the following variables. For all $k \in H^o$ and $t \in \mathcal{T}$,

$$h_k^t = \begin{cases} 1 \text{ if hub node } k \text{ is established at the beginning of the time period } t, \\ 0 \text{ otherwise.} \end{cases}$$

For all $k \in H^c$ and $t \in \mathcal{T} - \{T\}$,

$$h_k^t = \begin{cases} 1 \text{ if hub node } k \text{ is removed at the end of time period } t, \\ 0 \text{ otherwise.} \end{cases}$$

For all $k \in H^c$:

$$h_k^T = \begin{cases} 1 \text{ if hub node remains active until end of planning horizon,} \\ 0 \text{ otherwise,} \end{cases}$$

where T is the planning horizon length and y_{kl}^t is defined analogously for the set of edges.

2.3 Multi-Period Model (MPHLPPT)

The generality of the model is the same as single period case (i.e. HLPPT) and in addition we should have a linkage between periods and also express HLPPT in terms of newly defined sets and variables ((16) and (17)). Since the model becomes involved, therefore we explain the constraints step by step and make some labels (A, B, ...) to reach the final model by bringing these labels together.

Note 1 *Each edge is represented by $e \in E$ and corresponds to the edge (k, l). That means, k is the head and l is the tail node of that edge. Moreover, because the graph is assumed to be undirected, it is always assumed that $l > k$.*

(A): A closeable facility should either be closed before the last period or the corresponding variable takes 1 at the last period indicating that it has been open over the whole planning horizon. In Contrary, an openable facility can be opened just once. The following constraints are taking care of these facts for both types of facilities.

$$\sum_{t \in \mathcal{T}} h_k^t = 1, \qquad \forall k \in H^c, \tag{18}$$

$$\sum_{t \in \mathcal{T}} y_e^t = 1, \qquad \forall e \in E^c, \tag{19}$$

$$\sum_{t \in \mathcal{T}} h_k^t \leq 1, \qquad \forall k \in H^o, \tag{20}$$

$$\sum_{t \in \mathcal{T}} y_e^t \leq 1, \qquad \forall e \in E^o. \tag{21}$$

(B): It is assumed that there exists just a limited amount of resources (budget, labor and workforce, etc.) which only suffices to establish a limited number of facilities. For instance, for given $q1, q2 \in \mathbb{N}$, $q1$ hub nodes and $q2$ hub edges from among all the openable ones in each time period can be established.

$$\sum_{e \in E^o} y_e^t \leq q1, \qquad \forall t \in \mathcal{T}, \tag{22}$$

$$\sum_{k \in h^o} h_k^t \leq q2, \qquad \forall t \in \mathcal{T}. \tag{23}$$

(C): An openable hub edge between two end-points of k and l can have its end-points belonging to different sets of partitions. For all $t \in \mathcal{T}, l > k$,

- if k and l both are openable, an openable edge e can be opened if both end-point have been opened until now.

$$y_e^t \leq \sum_{t'=1}^{t} h_k^{t'}, \qquad \forall e \in E^o, k, l \in H^o,$$

$$y_e^t \le \sum_{t'=1}^{t} h_l^{t'}, \qquad \forall e \in E^o, k, l \in H^o.$$

- if $k(l)$ is closeable and $l(k)$ is openable, an openable edge e can be opened if the closeable endpoint is not closed yet and will not be closed afterwards; and the openable end-point is opened until beginning of this period.

$$y_e^t \le 1 - \sum_{t'=1}^{T-1} h_k^{t'}, \qquad \forall e \in E^o, k \in H^c, l \in H^o, \quad (24)$$

$$y_e^t \le \sum_{t'=1}^{t} h_l^{t'}, \qquad \forall e \in E^o, k \in H^c, l \in H^o, \quad (25)$$

$$y_e^t \le \sum_{t'=1}^{t} h_k^{t'}, \qquad \forall e \in E^o, k \in H^o, l \in H^c, \quad (26)$$

$$y_e^t \le 1 - \sum_{t'=1}^{T-1} h_l^{t'}, \qquad \forall e \in E^o, k \in H^o, l \in H^c. \quad (27)$$

- if k and l are closeable, an openable edge e can be opened if both end-points are not closed yet and will not be closed until the end of the $(T-1)$-th period.

$$y_e^t \le 1 - \sum_{t'=1}^{T-1} h_k^{t'}, \qquad \forall e \in E^o, k, l \in H^c, \quad (28)$$

$$y_e^t \le 1 - \sum_{t'=1}^{T-1} h_l^{t'}, \qquad \forall e \in E^o, k, l \in H^c. \quad (29)$$

- for a closeable edge $e \in E^c$, where in fact both end-points are closeable, e can remain open as long as both end-points remain open.

$$y_e^t \ge h_k^{t'}, \qquad \forall e \in E^c, k, l \in H^c,$$

$$y_e^t \ge h_l^{t'}, \qquad \forall e \in E^c, k, l \in H^c.$$

(D): For a given origin i and destination j, the flow from i to j can pass through the hub edge (k, l):

- if (k, l) is openable, it should have been opened until now,

$$x_{ijkl}^t + x_{ijlk}^t \le \sum_{t'=1}^{t} y_e^{t'}, \qquad \forall i, j > i, e \in E^o, t \in \mathcal{T}.$$

- if (k, l) is closeable, it should have not been closed yet.

$$x_{ijkl}^t + x_{ijlk}^t \le 1 - \sum_{t'=1}^{t-1} y_e^{t'}, \qquad \forall i, j > i, e \in E^c, t \in \mathcal{T}.$$

(E): For a given flow between two nodes when only one of them is a hub:

- if that hub node is openable, it should have been opened in a period since beginning of the planning horizon,

$$\sum_{l \neq k} x^t_{kjkl} \leq \sum_{t'=1}^{t} h^{t'}_k, \qquad \forall j,k \in H^o, k < j, t \in \mathcal{T},$$

$$\sum_{k \neq l} x^t_{ilkl} \leq \sum_{t'=1}^{t} h^{t'}_l, \qquad \forall i,l \in H^o, l > i, t \in \mathcal{T}.$$

- if that hub node was closeable, it should have not been closed yet.

$$\sum_{l \neq k} x^t_{kjkl} \leq 1 - \sum_{t'=1}^{t-1} h^{t'}_k, \qquad \forall j,k \in H^c, k < j, t \in \mathcal{T},$$

$$\sum_{l \neq k} x^t_{ilkl} \leq 1 - \sum_{t'=1}^{t-1} h^{t'}_l, \qquad \forall i,l \in H^c, l > i, t \in \mathcal{T}.$$

(F): The following constraints correspond to those of the single period model with the same modification as carried out with the other constraints.
For all $t \in \mathcal{T}$,

$$e^t_{ij} \leq \sum_{t'=1}^{t} |h^{t'}_i - h^{t'}_j|, \qquad \forall i,j \in H^o, j > i,$$

$$e^t_{ij} \leq 1 - |\sum_{t'=1}^{t} h^{t'}_i - \sum_{t'=1}^{t-1} h^{t'}_j|, \qquad \forall i \in H^o, j \in H^c, j > i,$$

$$e^t_{ij} \leq 1 - |\sum_{t'=1}^{t-1} h^{t'}_i - \sum_{t'=1}^{t} h^{t'}_j|, \qquad \forall i \in H^c, j \in H^o, j > i,$$

$$e^t_{ij} \leq \sum_{t'=1}^{t-1} |h^{t'}_i - h^{t'}_j|, \qquad \forall i,j \in H^c, j > i.$$

However, these constraints are not linear. In order to linearize them, we define more variables namely, $\delta^+_{ij} \geq 0$ and $\delta^-_{ij} \geq 0$.
For all $t \in \mathcal{T}$,

$$\sum_{t'=1}^{t} (h^{t'}_i - h^{t'}_j) = \delta^{t+}_{ij} - \delta^{t-}_{ij}, \qquad \forall i,j \in H^o, j > i,$$

$$1 - \sum_{t'=1}^{t} h^{t'}_i - \sum_{t'=1}^{t-1} h^{t'}_j = \delta^{t+}_{ij} - \delta^{t-}_{ij}, \qquad \forall i \in H^o, j \in H^c, j > i,$$

$$1-\sum_{t'=1}^{t-1} h_i^{t'} - \sum_{t'=1}^{t} h_j^{t'} = \delta_{ij}^{t+} - \delta_{ij}^{t-}, \qquad \forall i \in H^c, j \in H^o, j > i,$$

$$\sum_{t'=1}^{t-1} (h_i^{t'} - h_j^{t'}) = \delta_{ij}^{t+} - \delta_{ij}^{t-}, \qquad \forall i, j \in H^c, j > i,$$

and finally for all $t \in \mathcal{T}$, i and $j > i$ we add,

$$e_{ij}^t \leq \delta_{ij}^{t+} + \delta_{ij}^{t-} \leq 1.$$

(G): By definition, the variable a_{ijk} (b_{ijk}) is used to indicate whether there is a flow sent from (to) a spoke origin (destination) to (from) an arbitrary destination (origin) j (i) via a hub node k.
For all $t \in \mathcal{T}$,

$$a_{ijk}^t \leq 1 - \sum_{t'=1}^{t} h_i^{t'}, \qquad \forall i \in H^o, j > i, k \neq i, j,$$

$$a_{ijk}^t \leq \sum_{t'=1}^{t-1} h_i^{t'}, \qquad \forall i \in H^c, j > i, k \neq i, j,$$

$$b_{ijk}^t \leq 1 - \sum_{t'=1}^{t} h_j^{t'}, \qquad \forall j \in H^o, i < j, k \neq i, j,$$

$$b_{ijk}^t \leq \sum_{t'=1}^{t-1} h_j^{t'}, \qquad \forall j \in H^c, i < j, k \neq i, j.$$

(H): The flow emanating from a node i is received by a node j based on the status of i and j:
For all $t \in \mathcal{T}$,

$$a_{ijk}^t + \sum_{l \neq j,k} x_{ijlk}^t \leq \sum_{t'=1}^{t} h_k^{t'}, \qquad \forall i, j > i, k \in H^o, k \neq i, j,$$

$$a_{ijk}^t + \sum_{l \neq j,k} x_{ijlk}^t \leq 1 - \sum_{t'=1}^{t-1} h_k^{t'}, \qquad \forall i, j > i, k \in H^c, k \neq i, j,$$

$$b_{ijk}^t + \sum_{l \neq k,i} x_{ijkl}^t \leq \sum_{t'=1}^{t} h_k^{t'}, \qquad \forall i, j > i, k \in H^o, k \neq i, j,$$

$$b_{ijk}^t + \sum_{l \neq k,i} x_{ijkl}^t \leq 1 - \sum_{t'=1}^{t-1} h_k^{t'}, \qquad \forall i, j > i, k \in H^c, k \neq i, j,$$

$$e_{ij}^t + 2x_{ijij}^t + \sum_{l \neq j,i} (x_{ijil}^t + x_{ijlj}^t) \leq$$

$$\begin{cases} \sum_{t'=1}^{t}(h_i^{t'} + h_j^{t'}), & \forall i, j \in H^o, j > i, \\ \sum_{t'=1}^{t} h_i^{t'} + (1 - \sum_{t'=1}^{t-1} h_j^{t'}) & \forall i \in H^o, j \in H^c, j > i, \\ (1 - \sum_{t'=1}^{t-1} h_i^{t'}) + \sum_{t'=1}^{t} h_j^{t'}, & \forall i \in H^c, j \in H^o, j > i, \\ 2 - (\sum_{t'=1}^{t-1} h_i^{t'} + \sum_{t'=1}^{t-1} h_j^{t'}) & \forall i, j \in H^c, j > i. \end{cases}$$

Now, we can state MPHLPPT:

(MPHLPPT)

$$\begin{aligned}
Min \sum_{t \in \mathcal{T}} \Bigg(& \sum_i \sum_{j>i} \sum_k \sum_{l \neq k} \alpha^t (W_{ij}^t + W_{ji}^t) C_{kl}^t x_{ijkl}^t + \sum_i \sum_{j>i} (W_{ij}^t + W_{ji}^t) C_{ij}^t e_{ij}^t + \\
& \sum_i \sum_{j>i} \sum_{k \neq i,j} (W_{ij}^t + W_{ji}^t) C_{ik}^t a_{ijk}^t + \sum_i \sum_{j>i} \sum_{k \neq i,j} (W_{ij}^t + W_{ji}^t) C_{kj}^t b_{ijk}^t + \\
& \sum_{t \in \mathcal{T}} \sum_{k \in H^o} \left(F_k^t + \sum_{t'=t}^{T} HMC_k^{t'} \right) h_k^t + \sum_{t \in \mathcal{T}} \sum_{k \in H^c} \left(\sum_{t'=1}^{t} HMC_k^{t'} + HCC_k^t \right) {h_k}^t + \\
& \sum_{t \in \mathcal{T}} \sum_{e \in E^o} \left(I_e^t + \sum_{t'=t}^{\mathcal{T}} EMC_k^{t'} \right) y_e^t + \sum_{t \in \mathcal{T}} \sum_{k \in E^c} \left(\sum_{t'=1}^{t} EMC_e^{t'} + ECC_e^t \right) {y_e}^t \Bigg) \\
& - \sum_{k \in H^c} HCC_k^t h_k^T - \sum_{e \in E^c} ECC_e^t y_e^T \qquad (30)
\end{aligned}$$

$$\begin{aligned}
s.t. \; & \sum_{l \neq i} x_{ijil}^t + \sum_{l \neq i,j} a_{ijl}^t + e_{ij}^t = 1, && \forall t, i, j > i, \quad (31) \\
& \sum_{l \neq j} x_{ijlj}^t + \sum_{l \neq i,j} b_{ijl}^t + e_{ij}^t = 1, && \forall t, i, j > i, \quad (32) \\
& \sum_{l \neq k,i} x_{ijkl}^t + b_{ijk}^t - \sum_{l \neq k,j} x_{ijlk}^t - a_{ijk}^t = 0, && \forall t, i, j > i, k \neq i, j, \quad (33) \\
& \mathbf{A, C, E, F, G, H}, \\
& x_{ijkl}^t, a_{ijk}^t, b_{ijk}^t, e_{ij}^t \in (0,1), y_{kl}^t, h_k^t \in \{0,1\}.
\end{aligned}$$

The objective function sums up to the flow cost plus the setup, maintenance and closing costs. Obviously, there should not be any closing cost for the last period.

Constraints (31)-(33) are the flow conservation constraints. The rest of constraints take care of routing of the flows in each period by taking into account the statutory assumptions. (**A**) ensures that an openable facility will be opened at most once and a closeable facility will be closed at most once. However, closing will not happen in the last period and corresponding variables take 1 stating that it was working over the planning horizon. In (**C**), it is ensured that both end-points of a hub edge are hub nodes. Constraints (**D**) state that a flow in its path to destination if passes through more than one

hub node in the hub-level network, these nodes should be connected by hub edge(s). In (**E**) it is ensured that only a flow with an origin (a destination) of hub type is allowed to select a hub edge to depart from the origin (arrive to the destination). Constraints (**F**) check the end-points of spoke edges. Any flow from i to j, if enters to (depart from) a node other than i and j, that node must be a hub node. This is ensured by (**G**). Selection of edges on the path between origin and destination, i and j, depends on the status of i and j; whether both, none or just one of them is a node. This is checked by (**H**). In an uncapacitated environment, as also mentioned in [6], only hub node and hub edge variables might need to be considered as binary variables.
Obviously, if there would be no constraints on the number of facilities to be established at each iteration and, benefiting from using the facilities can dominate the cost incurred for setup and maintenance, most of hub facilities which are going to be opened will preferably be opened at the earlier periods. In this way, as soon as possible the economy of scale can be exploited. This tendency, is more sensible if the flow mass is homogeneously and monotonically increasing at each period. That is, at each stage there be more motivation of establishing hub-level facilities. But, if the flow in an area has very low density and dramatically increases in last periods, it is less likely for this area to receives hub facility in the earlier periods.
This issue is tackled in two ways; either by restricting the number of facilities that can be established at each period (by inclusion of **(B)** in the set of constraints) or by imposing some budget constraints.

3 Computational Results

A set of 10 initial configurations is generated for a randomly generated instance of size $n = 10$ (R10) so that their spatial layout are homogeneously distributed and are as scattered as possible with as small as possible of intersections.
In Figure 1 the layout of this instance is depicted. We divided the bounding area into four quarters and tried to choose the initial configurations as fair as possible. That is, every node appeared at least once as a hub node in an initial configuration and also the edges are selected from almost all parts of the layout.

The flows and costs are also randomly generated. Therefore, the cost as well as the flow, in general does not follow any structure.
The parameters of these instances are defined as follows,

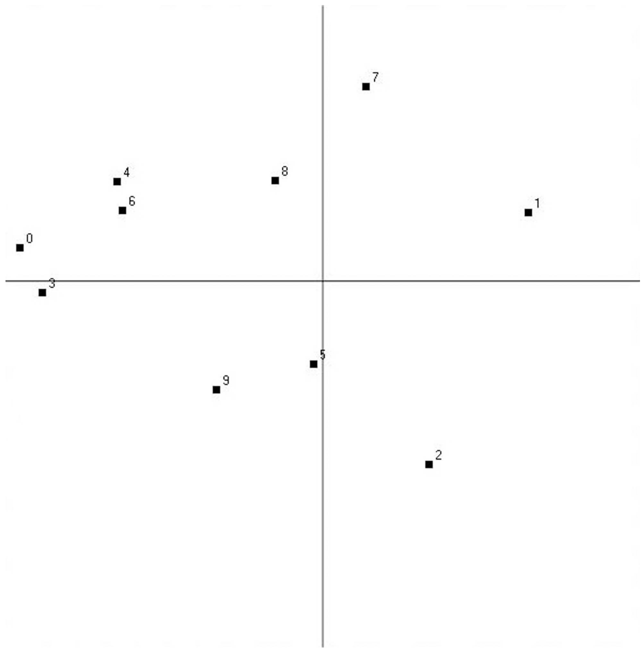

Fig. 1. Spatial layout of R10.

$$
\begin{aligned}
&F_i = 5000 && \forall i,\\
&HCC_i = 1000 && \forall i,\\
&HMC_i = 500 && \forall i,\\
&I_{ij} = 200 \times C_{ij} && \forall i, j > i,\\
&ECC_{ij} = 200 \times C_{ij} && \forall i, j > i,\\
&EMC_{ij} = 20 \times C_{ij} && \forall i, j > i.
\end{aligned}
$$

3.1 Constraints on the Number of Facilities to be Opened

The restriction on the number of facilities among the potential ones (hub nodes and hub edges) to be opened at each period can be drawn by the constraint set **(B)** ((22)-(23)).

3.1.1 Example

From our experiences, `CPLEX` is not capable of solving instances of larger than $n = 10$ with $T = 3$ in less than one week of computation before the variable reduction (before setting $W'_{ij} = W_{ij} + W_{ji}$ and exploiting symmetry in the paths and halving the number of variables as well as constraints). But afterwards, although the computational time drastically decreased, however, again it was not able to solve them in a reasonable amount of time (i.e. less than half a day for an instance of size 15 and $T = 3$). Therefore we report our results for a randomly generated instance of size 10 with 10 distinctly generated initial

Table 1
Constraints on number of nodes (CN).

CN	T_1. *Cpu(s)*	Root Node Gap(%)
{1-4}	168.27	2.21
{3-5},{5-8}	55.05	0.39
{1-6},{6-9}	93.24	2.49
{0-4},{4-8}	31.88	1.31
{1-5},{1-8}	66.31	3.47
{3-5},{5-7},{7-8}	70.36	0.45
{2-7},{0-7},{2-9}	161.86	4.68
{0-3},{3-5},{1-5}	47.11	0.62
{1-4},{1-2},{4-9}	116.53	2.10
{1-7},{3-7},{3-9}	95.44	4.56
Avg.	90.61	2.23

configurations. Computational results are reported in Table 1.
The computations are carried out on an Intel(R)Xeo, n(TM)CPU 2.60 GHz and 1 GB of RAM. Results are chosen from the best output of `CPLEX 9.1` and `CPLEX 11.0`.

Table 1, states that the root node gaps are small enough and in average the computational time is less than 2 minutes.
An interesting behavior of this problem is that, although the instances of size 10 with $T = 3$ (at least for our initial configurations) are mostly solved in less than 5 minutes. However, as the problem size grows from 10 to 15 the computational time dramatically increases. This clearly indicates the high complexity of the problem.

3.1.2 R15

In Figure 2, an optimally solved randomly generated instance of size 15, R15, is depicted. Here, we let $q1 = q2 = 3$. The computational time for solving this instance to the optimality using `CPLEX` is about 95513.47 seconds (more than 26 hours). The observed root node gap is 17.34% .

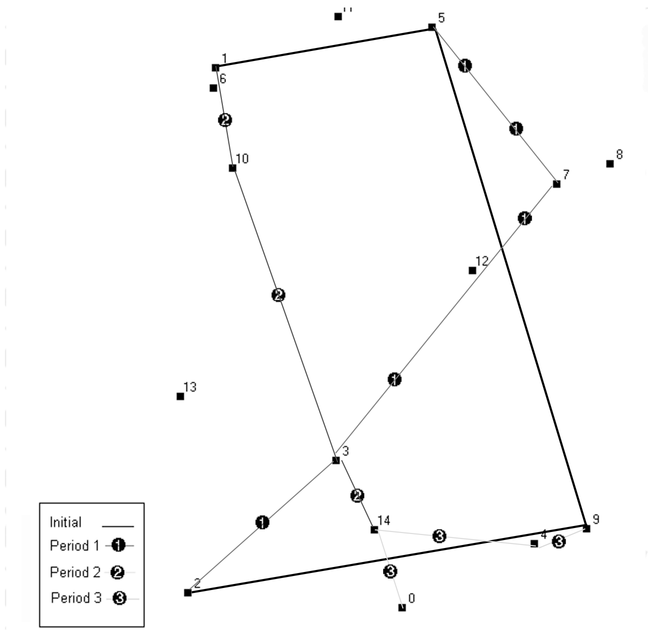

Fig. 2. CN: Solution to the R15.

3.2 Constrained by the Budgets for Activities (CB)

It has been assumed in (22)-(23) that there is a limited number of facilities that can be established at each period. It may not be only the financial aspects to prevent us. However, this can also be due to some other factors (like resources and machines, geographical or political issues, etc.). However, in our study only the financial issues are taken into account. Rather than directly restricting the number of facilities to be established, these constraints can be expressed in terms of amount of available budget for the activities of project at each period. This is in fact more realistic.

At each period, there exists a fixed amount of budget that can be invested for the facility establishment, maintenance and ceasing. Any capital available in a period but not invested then is subject to an interest rate and the returned value can be used in subsequent periods. This amount of return from the preceding period plus the fixed amount of budget considered for the current period sums up to the whole amount of available budget for this period.
New variables and parameters are defined as follows. For all $t \in T$:

B^t : the amount of initially available budget at the beginning of period t,
ρ^t : unit return factor on capital not invested in period t ($\rho^t > 1$),
η^t : remaining amount of budget at the end of a period t.

The amount of budget available the for example in the second period would amount to $B^2 + \rho^1\eta^1$.
According to the definition of these new variables, the following constraints are added to the model.

$$\sum_{k\in H^o}(F_k^1 h_k^1) + \sum_{k\in H^o}(h_k^1 \times HMC_k) + \sum_{k\in h^c}(HCC_k h_k^1) + \sum_{k\in h^c}(h_k^1 HMC_k)$$
$$+ \sum_{e\in E^o}(I_e^1 y_e^1) + \sum_{e\in E^o}(y_e^1 \times EMC_e) + \sum_{e\in y^c}(YCC_e y_e^1) + \sum_{e\in y^c}(y_e^1 YMC_e)$$
$$+\eta^1 = B^1$$
$$\sum_{k\in H^o}(F_k^t h_k^t) + \sum_{k\in H^o}((\sum_{t'=1}^{t} h_k^{t'}) \times HMC_k) + \sum_{k\in h^c}(HCC_k h_k^t)$$
$$+ \sum_{e\in E^o}(I_e^t y_e^t) + \sum_{e\in E^o}((\sum_{t'=1}^{t} y_e^{t'}) \times EMC_e) + \sum_{e\in h^c}(YCC_k y_e^t)$$
$$+ \sum_{k\in h^c}((1 - \sum_{t'=1}^{t-1} h_k^{t'}) \times HMC_k) + \sum_{e\in E^c}((1 - \sum_{t'=1}^{t-1} y_e^{t'}) \times EMC_e)$$
$$+\eta_t = B^t + (\rho^{t-1}\eta^{t-1}) \qquad \forall t = 2\ldots T-1$$
$$\sum_{k\in H^o}(F_k^T h_k^T) + \sum_{k\in H^o}((\sum_{t=1}^{T} h_k^t) \times HMC_k) + \sum_{k\in h^c}((1 - \sum_{t'=1}^{T-1} h_k^{t'}) \times HMC_k)$$
$$+ \sum_{e\in E^o}(I_e^T y_e^T) + \sum_{e\in E^o}((\sum_{t'=1}^{T} y_e^{t'}) \times EMC_e) + \sum_{e\in E^c}((1 - \sum_{t'=1}^{T-1} y_e^{t'}) \times EMC_e)$$
$$+\eta^T = B^T + (\rho^{T-1}\eta^{T-1})$$

3.2.1 Example

Again, we had a similar difficulty for solving instances of larger than 10 even with $T = 3$. Therefore we report our results for the same randomly generated instance with the same initial configurations as *CN*, $B^t = 200000$ and $\rho^t = 1.2$. Computational results are reported in Table 2.

In average, instances can be solved around 3 minutes and root node gaps are in general small.

3.2.2 R15

In Figure 3, R15 is solved for the *CB* case. Here, we let $T = 3$ and $B^t = 300000$ which is not a very tight budget capacity. To solve this instance, 47747.75 seconds (more than 13 hours) computational efforts is needed. The root node

Table 2
Constraints on number of nodes (CB).

CB	T_1. $Cpu(s)$	Root Node Gap(%)
{1-4}	801.28	25.46
{3-5},{5-8}	97.66	0.86
{1-6},{6-9}	140.99	1.87
{0-4},{4-8}	119.75	1.77
{1-5},{1-8}	170.49	5.78
{3-5},{5-7},{7-8}	71.09	2.64
{2-7},{0-7},{2-9}	349.91	2.81
{0-3},{3-5},{1-5}	75.69	1.82
{1-4},{1-2},{4-9}	119.85	5.19
{1-7},{3-7},{3-9}	89.00	1.83
Avg.	203.571	5.00

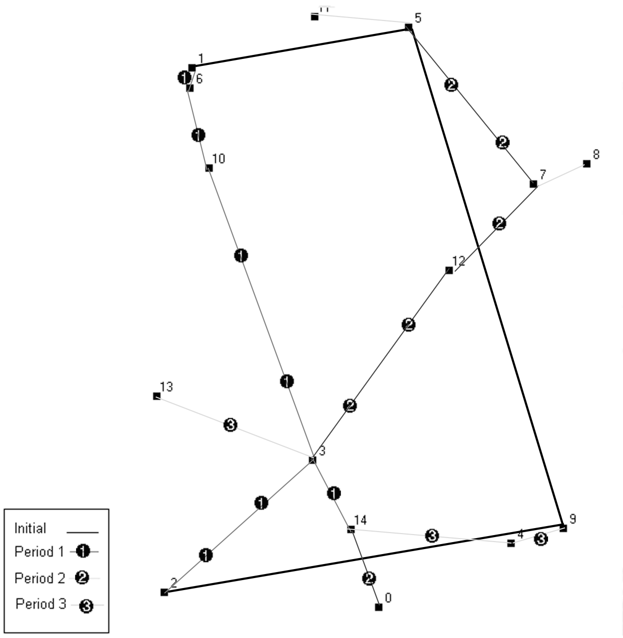

Fig. 3. CB: Solution to the R15.

gap shown to be 8.65%.

4 A Greedy Neighborhood Search for the MPHLPPT

Due to the high complexity of the MPHLPPT, solution of instances of the problem to optimality even for small size ones in a reasonable amount of time is not practically possible. Therefore we have to develop some heuristics which are capable of finding good solutions in reasonable amount of time. At the same time according to our experiences on instances of MPHLPPT and HLPPT, any complexity in the structure of such a heuristic can cause obvious inefficiencies of heuristic as the problem size grows. That is, our experiences show that incorporating any sort of short or long-term memories is not effective.

The heuristic algorithm that we are going to propose is a greedy neighborhood search. The key element of our algorithm is the neighborhood structure which determines rules to move from one configuration to another one. We adapt the neighborhood and adopt the idea of the *greedy*$^+$ algorithm proposed for HLPPT in [19], to solve instances of MPHLPPT. Again the search process explores the set of edges and the original neighborhood is proposed upon the edge features.

Definition 1 (Neighborhood Structure for MPHLPPT) *For a given period t and a given hub edge $i - j$, the neighborhood of this configuration will obey the following rules:*

- *if hub edge $i - j$ is a closeable hub edge:*
  - **"Close it from now on"***: if the hub edge is active now, then this hub is open since beginning of planning horizon and maybe after the current period. Thus, close it at the end of current period,*
  - **"Keep it open until now"***: if the hub edge is closed at this time, then it would be kept open from the beginning of planning horizon until now and will be closed at the end of current period until end of planning horizon,*
- *if the hub edge $i - j$ is an openable hub edge:*
  - **"Keep it closed until now"***: if the hub edge is active now then it should be kept closed until the end of this period and starts working from the successive period, if exists,*
  - **"Open it from now on"***: if the hub edge is closed at the current period then it becomes open from now on until end.*

In our idea, the described neighborhood structure is comprehensive enough to consider all possibilities of the solution space.
Now, we would like to translate our problem into the necessary components of a greedy algorithm.

- Set of all edges as the *set of candidates*,
- $\Delta = f^{new} - f^{cur}$ as the *selection function*,
- a functionality for checking the connectivity, to act as a *feasibility function* and,
- the objective function of MPHLPPT (to count the total cost: the hub-level network setup cost plus the rest of costs) as the *objective function*.

4.1 Algorithm

The proposed basic algorithm for the multi-period problem is drawn in Algorithm 1. We would like to note that two models have been proposed in the previous section. That is, once the number of facilities to be established in each period was restricted and another time, in a more realistic form, budget constraints were imposed. Both models share the same heuristic skeleton, except they have different *feasibility functions*. In the first model, the feasibility function is in charge of checking the connectivity and bounds on the number of established facilities and in the latter case, is responsible for avoiding violation of budget constraints.

Computational Results

It is observed that for the variants of MPHLPPT, problem instances up to size 10 can be solved to optimality in a reasonable amount of time. For the instances of size 15, almost all initial configurations that we have employed, could not reach to at least small gap solutions in less than half a day. In addition, our heuristic shown to be capable of finding optimal solution of instances of size $n = 5$ for almost any given initial configurations that is examined.
In this section we are going to solve instances of MPHLPPT for a variety of given and distinct initial configurations by our heuristic and compare results with those of `CPLEX`. Both variants of restrictions on the number of facilities to be established (CN) and the budget constrained (CB) are considered. Instances of MPHLPPT for both CN and CB, are solved by `CPLEX` to solutions with qualities similar to those of our heuristic solution.

From Table 3, one observes that, first of all the quality of the solutions of basic heuristic are quite satisfactory. Moreover, for a given solution quality, our heuristic outperforms `CPLEX` with respect to the time. This is also indicated in the last row Table 3. The average gap is 1.88% and in average such gap can be achieved by our heuristic 59 times faster than `CPLEX`. This can be visualized in the Figure 4.

Again, for CB, heuristic was capable of finding good solutions much faster than what `CPLEX` needs to find solutions with such qualities. The average gap is satisfactory and our heuristic was 62 times faster than `CPLEX`. Figure 5 gives an imagination of that.

Algorithm 1: A simple greedy algorithm for multi period HLPPT

```
Input: instance and init_conf
Output: x*
x̄ := x_initcfg;
min := Eval(x̄);
last_min := ∞;
repeated_min := 0;
while (repeated_min = 0) do
    f̄ := Eval(x̄);
    if f̄ ≤ min then
        min := f̄;
        x* := x̄;
    end
    foreach t = 1 to nrPeriods do
        foreach i = 1 to nrLocations * (nrLocations − 1)/2 do
            Δf := 0;
            x' := x̄;
            if i ∈ CloseableEdges then
                switch x'_i^t do
                    case 0: x'_i^t' := 1      ∀t' ≤ t ;
                    case 1: x'_i^t' := 0      ∀t' ≥ t;
                end
            else
                switch x'_i^t do
                    case 0  x'_i^t' := 1      ∀t' ≥ t ;
                    case 1  x'_i^t' := 0      ∀t' ≤ t;
                end
            end
            if is_not_feasible(x') then
                Δf := ∞;
            else
                Δf := Eval(x') − min;
            end
            if Δf < 0 then
                x* := x' ;
                min := Eval(x');
            end

        end

    end
    if min = last_min then
        repeated_min := repeated_min + 1;
    end
    last_min := min;
    x̄ = x*;
end
stop.
```

Table 3
Constraints on number of nodes (CN).

CN	CPLEX	Heuristic	
	T_c. $Cpu(s)$	T_1. $Cpu(s)$	Gap(%)
{1-4}	44.78	1.11	2.3
{3-5},{5-8}	55.52	1.19	0.1
{1-6},{6-9}	93.41	0.95	2.4
{0-4},{4-8}	30.67	1.19	0.7
{1-5},{1-8}	63.99	1.02	1.6
{3-5},{5-7},{7-8}	24.42	1.30	1.1
{2-7},{0-7},{2-9}	133.88	1.19	4
{0-3},{3-5},{1-5}	18.22	1.24	1.4
{1-4},{1-2},{4-9}	114.78	1.09	1.5
{1-7},{3-7},{3-9}	88.39	1.13	3.7
Avg.	66.80	1.14	1.88%

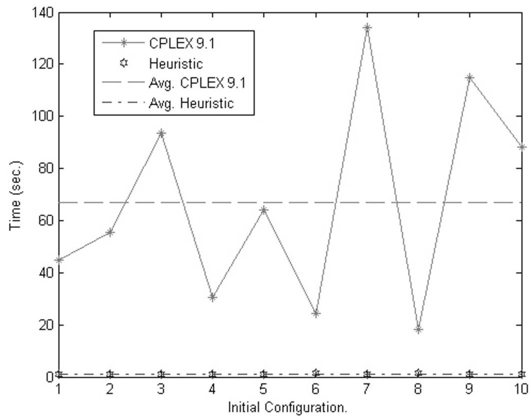

Fig. 4. CN: Heuristic vs. CPLEX

The results of Table 3 and Table 4 indicate that for the best-known solutions of our heuristic with the reported gaps in the last column, CPLEX finds a solution with the similar gap in a much higher amount of time. Although, the reported average gaps are satisfactory, however, they do exist. In the next Subsection we try to improve our solutions quality.

Local Search

From our observations out of the results of the greedy heuristic and considering the fact that complexity of the multi-period approach makes the like-hood of

Table 4
Constraints on number of nodes (CB).

CB	CPLEX	Heuristic	
	T_c. $Cpu(s)$	T_1. $Cpu(s)$	Gap(%)
{1-4}	303.93	1.34	5.7
{3-5},{5-8}	16.91	1.48	1.4
{1-6},{6-9}	140.55	1.74	0.1
{0-4},{4-8}	23.86	1.30	2
{1-5},{1-8}	112.13	1.31	1.6
{3-5},{5-7},{7-8}	51.44	1.64	1.3
{2-7},{0-7},{2-9}	111.83	1.53	6.7
{0-3},{3-5},{1-5}	71.63	1.81	0.6
{1-4},{1-2},{4-9}	73.41	1.45	4.4
{1-7},{3-7},{3-9}	16.81	1.42	7.4
Avg.	92.25	1.48	3.02

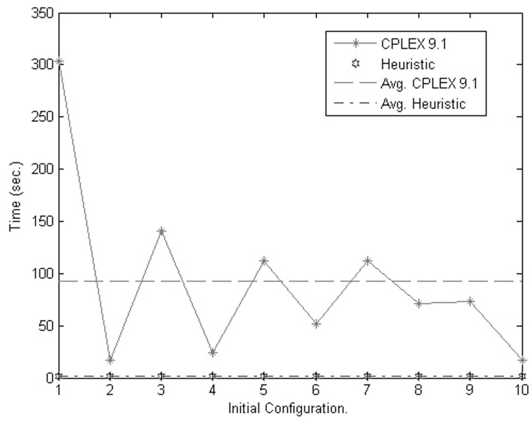

Fig. 5. CB: Heuristic vs. CPLEX

premature convergence to a local optimum in a greedy algorithm very high, we develop an additional approach in order to get rid of it, as much as possible.

Alternative Hub Edges

Out of visualizing the results, it has been observed that most of the time this local optimality is caused by inappropriate establishment of some facilities at periods. That is, though these facilities already exist in the optimal solution but they are not starting to work in the period appeared in the solution of greedy heuristic. At the same time, optimal solution is not achievable by a single move from current configuration based on the neighborhood rules and no more improvement is possible (because it can not be achieved by a single greedy move).

What can be beneficial to be accomplished here is to create trajectories in the search space to find better solutions. This can be done by closing those hub edges which are opened once in any period and trying to substitute them with those that have not been opened so far. That means, we give chance to the permanently spoke edges to become hub edges and then be subjected to the original neighborhood search, hoping to find better solutions. This may help the new configuration to find such a trajectory. A slight difference is that as soon as the first improvement is visited, the search moves to it rather than waiting for the best choice.

Procedure

In the sequel (Algorithm 2), the unfreezing process as explained earlier is displayed.

Algorithm 2: An improvement procedure for MPHLPPT

```
Input: y
Output: local_opt
local_opt := y;
min := ∞;
foreach i = 1  to  nrLocations − 1, j = i + 1  to  nrLocations do
    x := y;
    if hub edge i − j has been active once in a any period then
        x^t_ij := 0        ∀t;
        foreach p = i + 1  to  nrLocations do
            x^t_ip := 1        ∀t;
            x=Neighborhood_Search(x);
            if Eval(x) ≤ min then
                min := Eval(x);
                Local_Opt := x;
                y := x;
            end
        end
    else
        continue for the next hub edge;
    end
end
return local_opt.
```

After the instance name column, the first two columns in Table 5 and Table 6 are the results of heuristic before improvement. The second two are those after improvement and the last one is the CPLEX run-time for solutions with the smaller gaps as reported by heuristic after improvement.

Table 5
Constraints on number of nodes (CN).

CN	T_1. $Cpu(s)$	Gap(%)	T_1'. $Cpu(s)$	Gap(%)	CPLEX
{1-4}	1.11	2.3	16.78	0.7	151.66
{3-5},{5-8}	1.19	0.1	11.03	0.1	55.52
{1-6},{6-9}	0.95	2.4	8.13	2.4	93.41
{0-4},{4-8}	1.19	0.7	10.95	0.7	30.67
{1-5},{1-8}	1.02	1.6	25.03	0.9	63.99
{3-5},{5-7},{7-8}	1.30	1.1	24.36	0.00	71.55
{2-7},{0-7},{2-9}	1.19	4	25.19	0_{opt}	161.86
{0-3},{3-5},{1-5}	1.24	1.4	27.41	0_{opt}	47.11
{1-4},{1-2},{4-9}	1.09	1.5	12.47	1.4	114.78
{1-7},{3-7},{3-9}	1.13	3.7	12.69	3.7	88.39
Avg.	1.14	1.88	17.40	0.99	87.89

After improvement, as depicted in Table 5, the heuristic found optimal solution of some instances. In average, the gap is below one percent which is halved. Figure 6 visualizes the results.

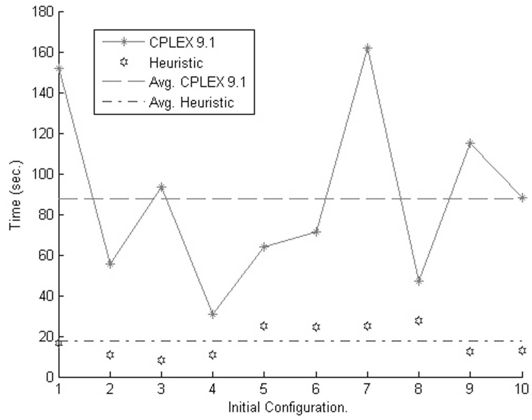

Fig. 6. CN: Heuristic vs. CPLEX after improvement.

R15

For the R15, our heuristic is capable of achieving a solution of CN with the gap of 2.00% in less than 120 seconds while `CPLEX` can not find a solution with the gap of less than 3.07% in less than 40878.81 seconds (around 340 times later). According to our results, `CPLEX` needs 95513.47 seconds (more than 26 hours) of computational efforts to solve this instance to optimality.

Table 6
Constraints on the amount of available budget (CB).

CB	T_1. *Cpu(s)*	Gap(%)	T_1'. *Cpu(s)*	Gap(%)	CPLEX
{1-4}	1.34	5.7	42.14	0_{opt}	801.28
{3-5},{5-8}	1.48	1.4	20.17	1.4	16.91
{1-6},{6-9}	1.74	0.1	24.03	0.1	140.55
{0-4},{4-8}	1.30	2	16.42	2	23.86
{1-5},{1-8}	1.31	1.6	31.72	1.1	132.57
{3-5},{5-7},{7-8}	1.64	1.3	57.28	1.2	58.17
{2-7},{0-7},{2-9}	1.53	6.7	48.72	2.2	137.08
{0-3},{3-5},{1-5}	1.81	0.6	15.80	0.6	71.63
{1-4},{1-2},{4-9}	1.45	4.4	83.22	0.4	113.85
{1-7},{3-7},{3-9}	1.42	7.4	19.44	7.4	16.81
Avg.	1.48	3.02	35.89	1.64	151.27

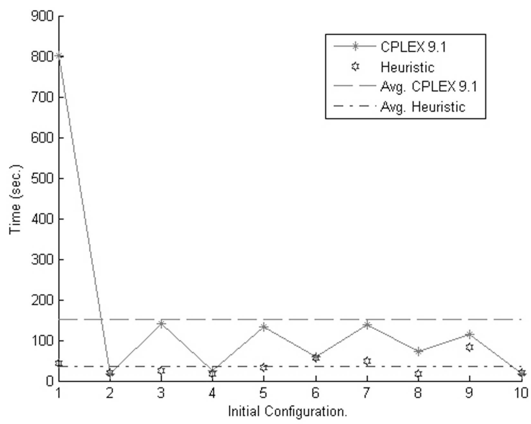

Fig. 7. CB: Heuristic vs. CPLEX after improvement.

As one can see in the Table 6, one instance reached to the optimality after improvements and most of them to a very small gap. The average gap reported in tables shows to become smaller after improvements (almost halved). In our experience, the best-known solution of our heuristic strongly depends on the Hamming distance between the spatial layout vector of the hub-level networks in the optimal solution and initial configuration of instance. Figure 7 visualizes the results.

R15

Again, for R15 with budget constraints, by means of our heuristic we could obtain a solution with the gap of 5.8% after 1104.60 seconds while a solution with such a gap could not be found earlier than 3383.39 seconds by `CPLEX` (3.06 times faster by heuristic).

Improved Algorithm

The improved algorithm is depicted in Algorithm 3.

A Larger Scale Instance

In this section we are going to solve larger instances of MPHLPPT for which no optimal solution is available. This gives us an impression of the run-time of our heuristic.
A set of randomly generated instances of size 40 with $T = 3, 6, 9$ and 12 are solved by our heuristic. The maintenance and ceasing costs are also considered in addition to the setup costs. Furthermore, the interest rate is set to $\alpha = 1.2$ in the budget constrained variant and the maximum number of hub facilities that can be setup in each period of CN is restricted to 3.

The initial configuration of our random instance of size 40 is depicted in Figure 8.

The run-times reported for this instance for both CN and CB are depicted in Figure 9

Summary and Conclusions

We proposed the first multi-period hub location problem model for application in public transport. In fact, we extended HLPPT [18] which shown to

Algorithm 3: A simple greedy algorithm for multi period HLPPT

```
Input: instance and init_conf
Output: x*
x̄ := x_initcfg;
min := Eval(x̄);
last_min := ∞;
repeated_min := 0;
while (repeated_min == 0) do
    f̄ := Eval(x̄);
    if f̄ ≤ min then
        min := f̄;
        x* = x̄;
    end
    foreach t = 1 to nrPeriods do
        foreach i = 1 to nrLocations * (nrLocations − 1)/2 do
            Δf := 0;
            x' := x̄;
            if i ∈ CloseableEdges then
                switch x'_i^t do
                    case 0: x'_i^{t'} = 1        ∀t' ≤ t ;
                    case 1: x'_i^{t'} = 0        ∀t' ≥ t;
                end
            else
                switch x'_i^t do
                    case 0 x'_i^{t'} = 1        ∀t' ≥ t ;
                    case 1 x'_i^{t'} = 0        ∀t' ≤ t;
                end
            end
            if is_not_feasible(x') then
                Δf := ∞;
            else
                Δf := Eval(x') − min;
            end
            if Δf < 0 then
                x* := x' ;
                min := Eval(x');
            end
        end
    end
    if min = last_min then
        repeated_min := repeated_min + 1;
        if repeated_min = 2 then
            goto 61;
        end
        if repeated_min = 1 then
            x̄= Alternate(Local_Opt);
            if Eval(x̄) ≤ min then
                min := Eval(x̄);
                Local_Opt := x;
                repeated_min := 0;
            else
                min := last_min;
            end
            goto 39;
        end
    end
    last_min := min;
    x̄ = x*;
end
stop.
```

be superior to other models for this application with the same assumptions. Some realistic assumptions, features and properties of real-life application are incorporated in this model. We have observed that even instances of very small size like 15 can not be solved in a reasonable amount of time, namely less than half a day. Therefore, a greedy neighborhood search heuristic equipped with the improvement methods is proposed. This heuristic shown to be promising and can reach to satisfactory solutions in much smaller amount of times when compared to the similar results by a standard solver.
In our future work, we will try to develop some Lagrangian heuristic to prepare some lower bounds for larger size instances. However, other ideas in the directions of upper bound heuristics will be examined. Incorporation of stochasticity in the system parameters to have more stable model is another

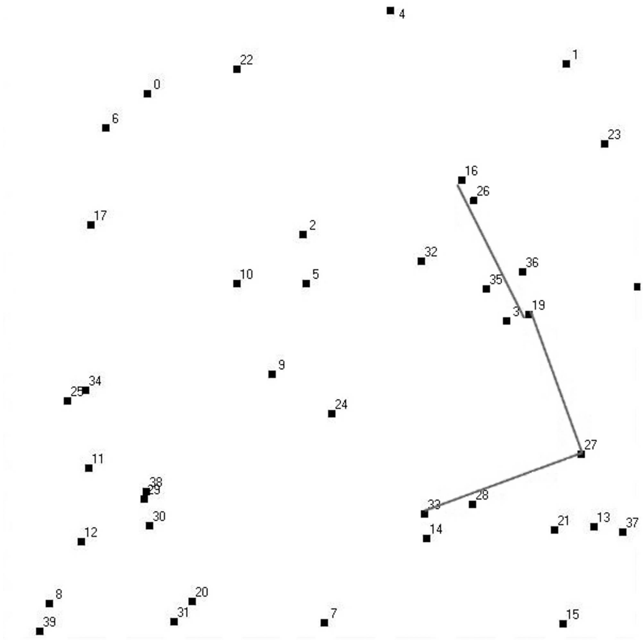

Fig. 8. Initial configuration for n=40

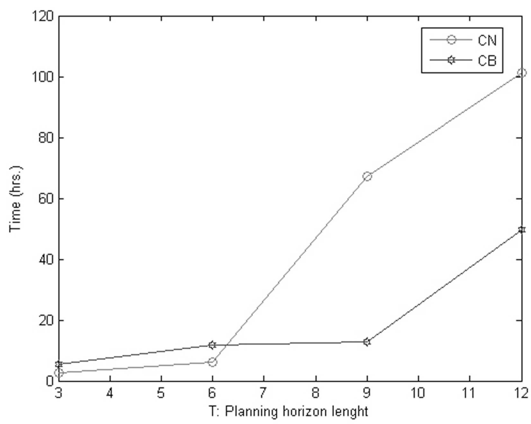

Fig. 9. Run-time in relation to the number of time periods.

direction for further studies.

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