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Vorwort

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Berichterstattung heißt hier Dokumentation des Transfers aktueller Ergebnisse aus mathematischer Forschungs- und Entwicklungsarbeit in industrielle Anwendungen und Softwareprodukte – und umgekehrt, denn Probleme der Praxis generieren neue interessante mathematische Fragestellungen.

Prof. Dr. Dieter Prätzel-Wolters
Institutsleiter

Kaiserslautern, im Juni 2001

ROBUST WORST-CASE OPTIMAL INVESTMENT

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Abstract. Based on a robustness concept adapted from mathematical statistics, we investigate robust optimal investment strategies for worst-case crash scenarios when the maximum crash height is not known a priori. We specify an efficiency criterion in terms of the certainty equivalents of optimal terminal wealth and explicitly solve the investor's portfolio problem for CRRA risk preferences. We also study the behavior of the minimax crash height and the efficiency of the associated strategies in the limiting case of infinitely many crashes.

Key Words: worst-case · crash scenario · robust optimization · Knightian uncertainty · efficiency · min-max approach

Mathematics Subject Classification (2010): 93E20, 91G10, 62C20, 62P05

1. Introduction

Worst-Case Optimality in Portfolio Selection. Continuous-time portfolio optimization is concerned with finding a trading strategy that maximizes expected utility from terminal wealth and/or consumption of an investor in a continuous-time financial market. The pioneering work in this area was done by Merton (1969, 1971) using methods from stochastic control theory, and recent years have seen significant

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progress in the field; we refer the reader to, e.g., the monographs Pham (2009) and Rogers (2013) for overviews of the subject.

In this paper, we focus on an important aspect that is neglected in the pure Merton-type setting: the presence of so-called crash scenarios as first introduced by Hua and Wilmott (1997). Their approach has been adapted for use in continuous-time portfolio optimization in Korn and Wilmott (2002). In addition, the traditional expected utility specification is replaced by a worst-case criterion similar to minimax criteria in game theory. More precisely, it is assumed that the total number and the maximum sizes of crashes are known in advance. In this setting, the investor maximizes expected utility from terminal wealth, assuming that the market will choose the worst possible crash times and worst sizes. Korn and Wilmott (2002) show existence and uniqueness of an optimal strategy for logarithmic utility. This strategy is characterized by the requirement that the investor is indifferent between the worst crash happening immediately and no crash happening at all. Generalizations of these results have been given in Korn and Menkens (2005) (more general utility functions), Korn and Steffensen (2007) (general dynamic programming approach), and Seifried (2010) (martingale approach based on controller-vs-stopper games).

As the maximum size of possible crashes is a crucial parameter that needs to be specified a priori in the worst-case portfolio optimization approach, this paper focuses on a robust approach that helps to deal with the uncertainty on the maximum crash height.

Robustness in Optimization, Statistics and Finance. The word ROBUST from Latin “robur, -is” originally means “of hard timber”, i.e., something that does not break easily. Mathematically, robustness is a stability notion and more specifically qualifies procedures as able to cope with a certain level of uncertainty in input data without producing uncontrollable output. Depending on the actual specification of input, output, and uncertainty, this concept has found many different areas of application within mathematics. A common theme in many of these approaches is a passage to worst-case or minimax solutions, and in this sense the worst-case approach outlined above is already inspired by the notion of robustness.

Robustness is particularly important in the context of *optimization*, or more specifically mathematical programming; see, e.g., the survey article Beyer and Sendhoff (2007) for a comprehensive account of this research. This paper is concerned with optimality in a robust context as well, albeit less in a mathematical programming framework, but rather in a stochastic control and finance setting. In addition, we focus on a particular criterion function by adopting the efficiency based approach to assess the uncertainty from a robust statistics context.

In *mathematical statistics*, many methods that are known to be optimal under ideal model conditions suffer from instabilities when confronted with situations where data may contain outliers. To address these issues, robust statistics has introduced distributional neighborhoods about an ideal model and developed concepts to quantify the sensitivity of procedures with respect to outliers; in addition, procedures that seek an optimal compromise between stability and efficiency have been provided.

In *finance* these ideas have been applied to capture uncertainty and robustness of risk functionals in, among others, Cont (2006), Föllmer, Schied, and Weber (2009), Cont, Deguest, and Scandolo (2010), Zähle (2013) and Krätschmer, Schied, and Zähle (2012). In this line of research, the desirable coherence property of risk functionals plays a crucial role. Coherence, however, requires a certain dominance condition of prior probability measures (compare eq. (4.4) in Föllmer, Schied, and Weber (2009)) in contrast to the usual outlier neighborhoods from robust statistics, and, similarly, in the context of robust utilities (compare eq. (4) in Schied (2005)). To be precise, their set $\mathcal{Q}$ of priors, which corresponds to the neighborhoods of robust statistics, is required to consist of measures dominated by a given reference probability and is used to obtain worst-case behavior as an infimum taken over $\mathcal{Q}$. By contrast, in worst-case robust portfolio optimization no such dominance condition is imposed. Finally, although both the CRRA utility used in the worst-case portfolio approach and standard quadratic loss used in robust statistics are unbounded, modified weak topologies as in Krätschmer, Schied, and Zähle (2012) and Zähle (2013) are not needed in the efficiency based robust approach. In addition, both robust statistics and worst-case portfolio selection rely on a concept of "nearness": in the former by quantifying the radii of neighborhoods, in the latter by specifying upper bounds for the number of crashes and the maximal crash size.

Robust Optimality and the RMX Approach. The common use of a nearness concept shows that in several respects the notion of optimality in robust statistics (see, e.g., Section 2.4 in Hampel, Ronchetti, Rousseeuw, and Stahel (1986) and Chapter 5 in Rieder (1994)) is in close analogy to worst-case portfolio optimization: The classical solutions—the maximum likelihood procedure (MLE) in statistics and the Merton portfolio problem in finance—are overly risky under non-idealized conditions such as crashes in portfolio optimization and outliers in statistics. In both cases, a robust approach makes it possible to tackle this instability.

Clearly, in either approach it is necessary to quantify the "distance" from the ideal situation, i.e., the maximal crash size and number of crashes or the radius of the relevant neighborhood, respectively. This distance can be regarded as a nuisance parameter, and a classical Bayesian approach would impose a prior distribution on this parameter. This, however, would presume prior knowledge. In the cases at hand, it is not clear how to specify an uninformative prior. In addition, this modeling approach would imply that, with sufficiently many observations, the relevant distance could eventually be estimated from data with arbitrary precision, which is not the case. Thus, in contrast to the probabilistic Bayesian model, in our approach we suppose that the parameters are subject to *Knightian uncertainty* in the sense of Knight (1921) and consequently do not impose any distributional assumptions.

In statistics, Rieder, Kohl, and Ruckdeschel (2008) have successfully addressed such issues using an additional layer of robustness that provides a rationale for selecting this distance when it is not known: The basic idea is to measure relative performance of a procedure which does not know the radius against the "oracle strategy" that does know the true radius. This approach leads to the notion of asymptotic relative efficiency (ARE) in statistics: For each candidate procedure, one determines its individual least favorable situation among all possible admissible radii, and then selects the procedure that attains the best worst-case behavior. This is defined as the rmx procedure (for *radius minimax*) RMXE. Denoting by $S(r)$ the optimal procedure for radius r and the neighborhood of radius r by $\mathrm{nbd}(r)$, this amounts to considering the efficiency quotient

$$q(r', r) = \mathrm{maxMSE}(S(r'), \mathrm{nbd}(r)) \big/ \mathrm{maxMSE}(S(r), \mathrm{nbd}(r)) \tag{1.1}$$

and allows us to define the rmx procedure $S(r^*)$ where r^* is chosen such that

$$\inf_r q(r^*, r) = \sup_{r'} \inf_r q(r', r).$$

In fact, retrospectively, this approach could be seen as a *local Savage Minimax Regret approach*, complementing the robustification provided by Gilboa and Schmeidler (1989) by a local aspect—local, because the "distance" to the ideal conditions is minimax-ed. The performance of the rmx approach is illustrated in Appendix B.

Scope of the Efficiency Quotient Approach. The above formulation of the efficiency quotient is tailored to address a specific statistical question. However, the notion of efficiency is easily and naturally transferred to more general situations that involve a parameter that specifies an unknown "distance" from ideal conditions. In this paper we will use the efficiency quotient approach to find robustly optimal trading strategies under the threat of a crash of unknown size.

Organization of the Paper. The remainder of this article is organized as follows: In Section 2 we introduce the financial market model. To motivate the use of the efficiency quotients in a financial context, we first detail a simple example with unknown excess returns and then introduce the general robust optimality criterion in Section 3. In Section 4 we identify the relevant worst-case scenarios for misspecified crash sizes. Based on this analysis, we are able to determine robust worst-case optimal strategies in Section 5. Section 6 generalizes our results to settings with multiple crashes and investigates the behavior of the optimal solutions for a growing number of crashes. While the optimality results of Sections 5 and 6 establish optimality within the class of worst-case strategies, Section 7 extends optimality to the wider class of arbitrary admissible strategies. Section 8 concludes and points towards possible extensions. The necessary mathematical results and proofs are gathered in Appendix A, and Appendix B provides an illustration of rmx procedures in robust statistics.

2. Financial Market and Non-Robust Portfolio Optimization

Financial Market Model and Crash Scenarios. The financial market consists of a riskless money market account and a risky stock with dynamics

$$\begin{aligned} \mathrm{d}P_0(t) &= P_0(t)\, r\, \mathrm{d}t \\ \mathrm{d}P_1(t) &= P_1(t)\left[(r+\lambda)\mathrm{d}t + \sigma\, \mathrm{d}W(t)\right], \quad P_1(0) = p_1 \end{aligned} \tag{2.1}$$

where r, λ, σ are positive constants. In addition, there can be finitely many (k, say) market crashes, modeled as stopping times $\tau^{(1)}, \dots, \tau^{(k)}$. At each time $t = \tau^{(i)}$ the stock may drop by up to a fraction $\ell \in [0, 1]$ of its value. Thus in the crash scenario $\tau^{(i)}, \ell$ we have

$$P_1(\tau^{(i)}) = (1-\ell)P_1(\tau^{(i)}-).$$

Crucially, there are no distributional assumptions on the crash times $\tau^{(i)}$: The crash times are subject to Knightian uncertainty.

If the investor is able to specify the maximal number k of possible market crashes and the size of the maximum crash height ℓ, the associated worst-case optimal investment problem with *constant relative risk aversion* (CRRA) utility has been analyzed by Korn and Steffensen (2007) and Seifried (2010). By contrast, if the investor is unsure how many crashes may occur and which sizes they may have—i.e., the maximal number of crashes and their *maximum sizes* ℓ are themselves subject to uncertainty—this literature offers no guidance.

Wealth Dynamics. The crash is unknown a priori, but can be observed when it occurs. The investor's strategy can thus be specified by predictable processes $\pi = (\pi^{(0)}, \dots, \pi^{(k)})$ where $\pi^{(i)}(t)$ represents the fraction of wealth invested into the risky asset at time t with i crashes outstanding (equivalently, when $j = k+1-i$ have occurred). His wealth dynamics are given by

$$\begin{aligned} \mathrm{d}X^\pi(t) &= X^\pi(t)\left[(r + \pi^{(j)}(t)\lambda)\,\mathrm{d}t + \pi^{(j)}(t)\sigma\,\mathrm{d}W(t)\right] \text{ on } [\tau^{(i-1)}, \tau^{(i)}) \\ X^\pi(\tau^{(i)}) &= (1 - \pi^{(j)}(\tau^{(i)})\ell)X^\pi(\tau^{(i-1)}-), \quad X_0^\pi = x \end{aligned}$$

where $i = 1, \dots, k+1$, $\tau^{(0)} \triangleq 0$, $\tau^{(k+1)} \triangleq \infty$. Note that the strategy $\pi^{(j)}$ is valid from $\tau^{(i-1)}$ up to and including $\tau^{(i)}$. The portfolio strategy π is said to be *admissible* if $\pi^{(j)}(t) \in [0,1]$ for all $t \geq 0$, and we denote by $\mathcal{A}^{(k)}$ the class of all admissible portfolio strategies.

As a consequence, the investor avoids bankruptcy in a crash for every $\pi \in \mathcal{A}^{(k)}$, because then $\ell\pi_t^{(j)} \leq 1$ for all $t \geq 0$, $\ell \in [0,1]$. Here and in the following, we write X^π instead of $X^{\pi,\tau^{(1)},\dots,\tau^{(k)}}$ for ease of notation.

Investor Preferences towards Risk and Uncertainty. The investor's attitudes towards risk are modeled by a classical CRRA utility function. By contrast, as to the uncertainty implied by the presence of market crashes, he takes a worst-case approach. Thus, if k and ℓ are known, his goal is to maximize expected utility for the worst possible crash-scenarios over all investment strategies, i.e.,

$$\sup_{\pi \in \mathcal{A}^{(k)}} \inf_{\tau^{(1)},\dots,\tau^{(k)}} \mathbb{E}\left[u(X^\pi(T))\right]$$

where $u(x) = x^{1-\gamma}/(1-\gamma)$ and $\gamma > 0$, $\gamma \neq 1$ is the investor's relative risk aversion. Equivalently, his goal is to maximize the worst-case certainty equivalent of terminal wealth, i.e.,

$$\sup_{\pi \in \mathcal{A}^{(k)}} \inf_{\tau^{(1)},\dots,\tau^{(k)}} \mathrm{CE}(X^\pi(T)) \tag{2.2}$$

where $\mathrm{CE}(X) \triangleq u^{-1}(\mathbb{E}[u(X)])$. Note that while $\mathbb{E}[u(X)]$ is measured on a utility scale, $\mathrm{CE}(X)$ is a monetary (say, dollar) value.

Remark 2.1. The case of unit risk aversion (i.e., $u(x) = \ln(x)$ and $\mathrm{CE}(X) = \exp\{\mathbb{E}[\ln(X)]\}$) is obtained in the limit $\gamma \to 1$. We do not consider this specification separately since the analysis is analogous to (but simpler than) that of the case $\gamma \neq 1$. $\diamond$

In the situation without crashes, i.e., when no further crashes can occur, the corresponding optimal admissible strategy is just the Merton strategy π^M,

$$\pi^M(t) = \lambda/(\gamma\sigma^2).$$

We assume throughout this article that the market and risk aversion parameters λ, σ and γ are such that $\lambda \leq \gamma\sigma^2$, i.e., such that $\pi^M \in \mathcal{A}^{(k)}$.

3. Robust Optimization in the Merton Model

In the following we investigate the optimal portfolio problem (2.2) when the maximum crash heights ℓ are not known a priori. Before we provide the general definition of the robust optimization criterion, we motivate our approach by reconsidering a classical problem and augmenting it with model uncertainty.

Uncertain Excess Return in the Merton Model. We consider the portfolio optimization problem in the market model (2.1) in the absence of crashes (formally, $k = 0$), but with the additional feature that the stock excess return λ is an *unknown* constant in $[0, \lambda_{\max}]$. If the investor believes that $\lambda = \lambda'$, he will use the associated Merton strategy $\pi^M = \lambda'/(\gamma\sigma^2)$. For a possibly time-dependent deterministic strategy π, a straightforward calculation yields the expected utility

$$\mathbb{E}[u(X^\pi(T))] = u(x) \exp\left\{(1-\gamma)\int_0^T \left[r + \pi(t)\lambda - \tfrac{1}{2}\gamma\pi(t)^2\sigma^2\right] \mathrm{d}t\right\}$$

and the corresponding certainty equivalent

$$\mathrm{CE}(X^\pi(T)) = x \exp\left\{\int_0^T \left[r + \pi(t)\lambda - \tfrac{1}{2}\gamma\pi(t)^2\sigma^2\right] \mathrm{d}t\right\}. \tag{3.1}$$

In particular, for a possibly misspecified excess return λ' we obtain

$$w(\lambda', \lambda) \triangleq \mathrm{CE}(X^{\pi^M}(T)) = x \exp\left\{\left[r + \tfrac{\lambda'\lambda - (\lambda')^2/2}{\gamma\sigma^2}\right] T\right\} \tag{3.2}$$

where λ denotes the "true" value of the excess return. On the other hand, the *optimal* dollar performance that would be attainable in an ideal world without parameter uncertainty is given by $w(\lambda, \lambda)$. Now define the efficiency $q(\lambda', \lambda)$ as the *fraction* of optimal dollar performance attained with the misspecified model parameter $\lambda = \lambda'$, i.e.,

$$q(\lambda', \lambda) \triangleq w(\lambda', \lambda) \,/\, w(\lambda, \lambda). \tag{3.3}$$

The investor's aim is to maximize efficiency in the most adverse parameter set. Thus he seeks a robustly optimal strategy $\lambda^\star$ to

$$\text{maximize} \quad \inf_{\lambda \in [0, \lambda_{\max}]} q(\lambda', \lambda) \text{ over all excess return estimates } \lambda'. \tag{3.4}$$

Plugging (3.2) into the criterion (3.3), we obtain

$$q(\lambda', \lambda) = \exp\left\{-\tfrac{T}{2\gamma\sigma^2}(\lambda' - \lambda)^2\right\}. \tag{3.5}$$

Hence for the simple Merton model the robustly optimal parameter estimate in (3.4) is given by

$$\lambda^\star = \tfrac{1}{2}\lambda_{\max}.$$

In Figure 1 we illustrate the efficiency criterion (3.5) as a function of λ where we have set $\gamma = 1$, $T = 10$ and $\sigma = 0.40$. In the left display, the unknown parameter λ varies in $[0, 1]$, while in the right it varies in $[0, 0.5]$. Note that in both cases the boundary values of $q(\lambda', \lambda)$

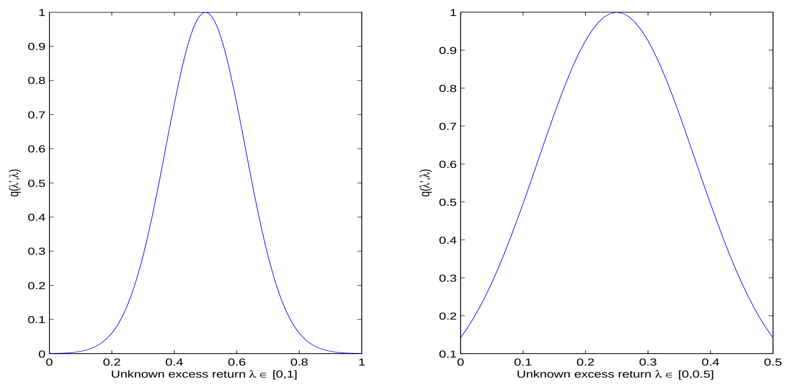

FIGURE 1. Efficiency $q(\lambda', \lambda)$ for $\lambda \in [0, 1]$ with $\lambda' = 0.5$ (LHS) and for $\lambda \in [0, 0.5]$ with $\lambda' = 0.25$ (RHS).

coincide. We will show below that this is a general feature of robustly optimal strategies.

Robust Optimality Criterion: General Definition. In the context of a general stochastic control problem, suppose that using the parameter specification θ' the agent attains a monetary certainty equivalent $w(\theta', \theta)$ in the model with "true" parameter θ. Let Θ denote the set of all possible parameter values. Then, as above, the efficiency $q(\theta', \theta)$ is defined via

$$q(\theta', \theta) \triangleq w(\theta', \theta) \,/\, w(\theta, \theta). \tag{3.6}$$

Note that since both $w(\theta', \theta)$ and $w(\theta, \theta)$ represent dollar values, this is an economically meaningful concept: The efficiency represents the fraction of the optimal dollar value that is attained with a particular strategy. The general robust optimization problem is to

$$\text{maximize } \inf_{\theta \in \Theta} q(\theta', \theta) \text{ over all } \theta' \in \Theta. \tag{P}$$

REMARK 3.1. The maximization in (P) extends over parameter specifications, not over strategies. Instead of maximizing the worst-parameter performance over *all* admissible strategies, it makes intuitive sense to concentrate on strategies that are *optimal for at least one parameter specification*. However, from a purely mathematical perspective it is not clear at this stage whether by passing from the class of worst-case optimal strategies to more general admissible strategies a superior worst-case efficiency could be attained. We show rigorously in Section 7 that, indeed, every admissible strategy is dominated by some worst-case optimal strategy. Therefore the robustly optimal strategy is optimal among *all* admissible strategies. $\diamond$

REMARK 3.2. The definition of efficiency in (3.6) is based upon certainty equivalents, not utility values. Since utilities are unique only up to affine transformations, quotients of utilities are in general not a well-defined concept. As an alternative to (3.6), it might also be interesting to study the efficiency criterion

$$\tilde{q}(\theta', \theta) \triangleq w(\theta', \theta) - w(\theta, \theta).$$

Clearly, this specification appears particularly suitable for problems with CARA risk preferences, but may also yield valuable insights for problems with CRRA utility. ◇

4. Worst-Case Scenarios

In this article, we consider worst cases on two levels, i.e., as to crash times and as to crash sizes. To distinguish these levels, in the following, we use the term *least favorable* for the worst case with regard to the crash sizes and *worst-case* for unknown crash-times.

In this section and the next we investigate optimal investment for worst-case crash scenarios with an uncertain maximal crash size ℓ. We assume that there can be at most one crash ($k = 1$); the case $k > 1$ is addressed in Section 6. We identify the worst-case crash scenarios for alternative candidate strategies and parameter specifications and determine the associated performance.

Notation. In the following we fix $\ell, \beta \in [0, 1]$. ℓ represents the "true" maximal crash size, whereas β is the crash size assumed by the investor in a possibly misspecified model. With a slight abuse of notation we denote by $\pi^{(\ell)} = \pi^{(1,\ell)}$ the optimal pre-crash investment strategy for worst-case crash scenarios of maximal size ℓ (obtained for ℓ to be known). $\pi^{(\ell)}$ is uniquely determined via an ordinary differential equation; see Korn and Steffensen (2007) or Seifried (2010). More precisely, we have

$$\frac{\mathrm{d}\pi^{(\ell)}(t)}{\mathrm{d}t} = \left(\tfrac{1}{\ell} - \pi^{(\ell)}(t)\right) \left\{ \lambda \pi^{(\ell)}(t) - \tfrac{1}{2} \left[\gamma \sigma^2 \pi^{(\ell)}(t)^2 + \tfrac{\lambda^2}{\gamma \sigma^2} \right] \right\} \tag{4.1}$$

subject to the boundary condition $\pi^{(\ell)}(T) = 0$. As shown by Seifried (2010), the worst-case optimal strategy $\pi^{(\ell)}$ is characterized by an indifference property: If the investor implements the strategy $\pi^{(\ell)}$, then his utility is independent of the timing of a crash with maximal size ℓ. In that sense, at the optimum the investor is indifferent concerning the crash. This is referred to below as the *indifference-optimality principle*.

We now determine the performance $w(\beta, \ell) \triangleq w^{(\ell)}(x, \pi^{(\beta)})$ of the strategy $\pi^{(\beta)}$ in the worst-case crash scenario with crash size ℓ.

Worst-Case Crash Scenario for $\ell < \beta$. For $\ell < \beta$ the investor assumes a larger maximal crash size than is possible in the true model. Thus he errs on the side of caution, and the strategy $\pi^{(\beta)}$ is too conservative. In fact, equation (4.1) implies that $\pi^{(\beta)}(t) < \pi^{(\ell)}(t)$ for all $t \in [0, T]$, as will be shown for the more general case of finitely many crashes in Corollary A.2 below. This implies that the investor is overinsured and that the strategy $\pi^{(\beta)}$ would only fare better in a crash. Hence the worst case is the no-crash scenario. It follows from equation (3.1) that the worst-case performance of $\pi^{(\beta)}$ is given by

$$w(\beta, \ell) = x \exp\left\{ \int_0^T \left[r + \pi^{(\beta)}(t)\lambda - \tfrac{1}{2}\gamma \pi^{(\beta)}(t)^2\sigma^2 \right] \mathrm{d}t \right\}. \tag{4.2}$$

Worst-Case Crash Scenario for $\ell \geq \beta$. For $\ell \geq \beta$ the actual crash size is potentially larger than anticipated by the strategy $\pi^{(\beta)}$. Since with $\beta \leq \ell$ we have $\pi^{(\beta)}(t) \geq \pi^{(\ell)}(t)$, and the strategy $\pi^{(\beta)}$ is too risk-prone. Thus the investor is underinsured against a crash of size $\ell > \beta$, and the worst case is an immediate crash of maximal size ℓ. The associated worst-case performance is

$$w(\beta, \ell) = x(1 - \pi^{(\beta)}(0)\ell) \exp\left\{ [r + \tfrac{1}{2}\tfrac{\lambda^2}{\gamma\sigma^2}]T \right\}. \tag{4.3}$$

Note that due to our assumptions we have that $\pi^{(\beta)}(0) \leq \pi^M < 1$ and thus the worst-case performance is well-defined.

5. Efficiency

This section provides a solution to the worst-case portfolio problem that is robust with respect to uncertainty in the maximal possible crash size.

Efficiency Criterion. In view of the results of Section 4 we are in a position to evaluate the efficiency criterion (3.6) for the worst-case portfolio problem with uncertain crash size,

$$q(\beta, \ell) \triangleq w(\beta, \ell) \,/\, w(\ell, \ell).$$

For notational convenience, we rescale q in a monotone way via $f(x) = \ln(x) + \frac{1}{2}\frac{\lambda^2}{\gamma\sigma^2}T$. Using (4.2) and (4.3) we then obtain

$$q(\beta, \ell) = \begin{cases} \int_0^T \left[\pi^{(\beta)}(t)\lambda - \frac{1}{2}\gamma\pi^{(\beta)}(t)^2\sigma^2 \right] \mathrm{d}t - \ln(1 - \pi^{(\ell)}(0)\ell), & \ell < \beta \\ \ln(1 - \pi^{(\beta)}(0)\ell) - \ln(1 - \pi^{(\ell)}(0)\ell) + \frac{1}{2}\frac{\lambda^2}{\gamma\sigma^2}T, & \ell \geq \beta. \end{cases} \tag{5.1}$$

Note that the initial wealth $x > 0$ cancels. The respective worst-case efficiencies are illustrated in Figure 2.

Analysis of Local Minima. The next step is to investigate the function $\ell \mapsto q(\beta, \ell)$ for a fixed value of β to identify the parameter values for ℓ that produce the least favorable efficiency for a given strategy $\pi^{(\beta)}$. By Lemma A.9 the minimum of $\ell \mapsto q(\beta, \ell)$ is attained either at $\ell = 0$ or at $\ell = 1$. Hence it suffices to consider $q(\beta, \ell)$ for $\ell = 0, 1$. In addition, by Corollary A.7, the functions $\beta \mapsto q(\beta, \ell)$, $\ell = 0, 1$, are continuous.

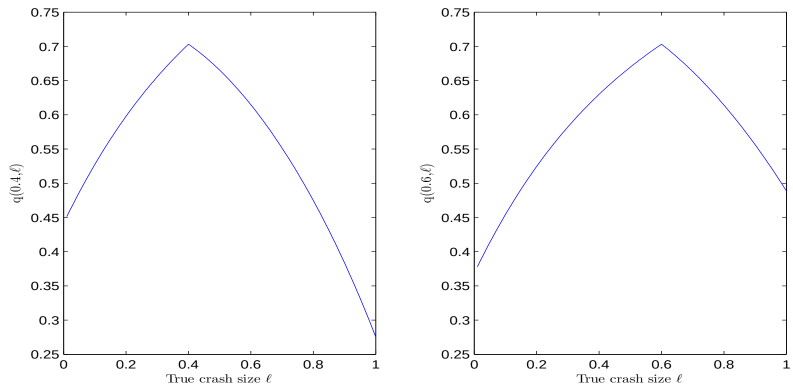

FIGURE 2. Efficiency $q(\beta,\ell)$ for $\beta = 0.4$ (LHS) and $\beta = 0.6$ (RHS) for fixed parameters $\mu = 0.20$, $r = 0.05$, $\sigma = 0.40$, $T = 10$ and $\gamma = 1$.

We set

$$\begin{aligned}\mu_0(\beta) &\triangleq q(\beta,0) = \int_0^T \left[\pi^{(\beta)}(t)\lambda - \tfrac{1}{2}\gamma\pi^{(\beta)}(t)^2\sigma^2\right] \mathrm{d}t\\ \mu_1(\beta) &\triangleq q(\beta,1) = \ln(1-\pi^{(\beta)}(0)) - \ln(1-\pi^{(1)}(0)) + \tfrac{1}{2}\tfrac{\lambda^2}{\gamma\sigma^2}T.\end{aligned}$$

Since $\pi^{(\beta)}(t) < \pi^{(\ell)}(t)$ for $\ell < \beta$, it follows that

$$\begin{aligned}&\mu_0 \text{ is decreasing in } \beta,\\ &\mu_0(0) = \tfrac{1}{2}\tfrac{\lambda^2}{\gamma\sigma^2}T,\\ &\mu_0(1) = \int_0^T \left[\pi^{(1)}(t)\lambda - \tfrac{1}{2}\gamma\pi^{(1)}(t)^2\sigma^2\right] \mathrm{d}t\\ &\qquad\;\; = \tfrac{1}{2}\tfrac{\lambda^2}{\gamma\sigma^2}T - \ln(1-\pi^{(1)}(0))\end{aligned}\tag{5.2}$$

where the last identity is due to the indifference-optimality principle. Similarly, concerning the second local minimum we have

$$\begin{aligned}&\mu_1 \text{ is increasing in } \beta,\\ &\mu_1(0) = \ln(1-\pi^M) - \ln(1-\pi^{(1)}(0)) + \tfrac{1}{2}\tfrac{\lambda^2}{\gamma\sigma^2}T,\\ &\mu_1(1) = \tfrac{1}{2}\tfrac{\lambda^2}{\gamma\sigma^2}T.\end{aligned}\tag{5.3}$$

Robust Worst-Case Optimal Strategy. By (5.2), (5.3) and continuity of μ_0 and μ_1 it follows that there exists a unique intersection point $\beta^\star \in [0,1]$ of μ_0 and μ_1,

$$\mu_0(\beta^\star) = \mu_1(\beta^\star).\tag{5.4}$$

Thus $\beta^\star$ balances the performances in the two most adverse scenarios $\ell = 0$ and $\ell = 1$. Figure 3 displays the intersection of the curves μ_0 and μ_1. Returning to the robust worst-case portfolio problem, as a consequence of Corollary A.7 and Lemma A.9, we record the following proposition.

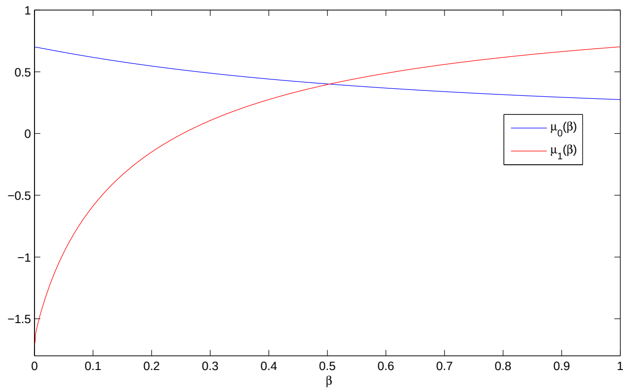

FIGURE 3. Left local minimum $\mu_0(\beta)$ and right local minimum $\mu_1(\beta)$ of $\ell \mapsto q(\beta,\ell)$ for fixed market parameters $\mu = 0.20$, $\sigma = 0.40$, $r = 0.05$, time horizon $T = 10$ and risk aversion $\gamma = 1$.

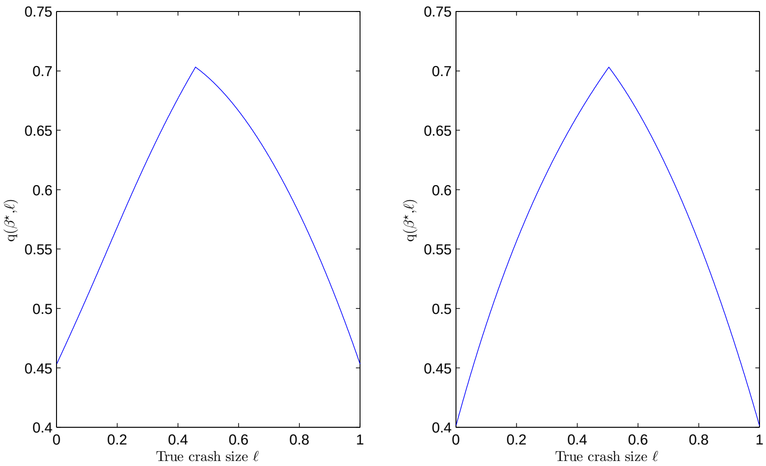

FIGURE 4. Least favorable performance $q(\beta,\ell)$ as a function of ℓ (market parameters: $\mu = 0.20$, $\sigma = 0.40$, $r = 0.05$, $T = 10$) with robustly optimal $\beta^\star = 0.458$ for $\gamma = 0.5$ (LHS) and $\beta^\star = 0.504$ for $\gamma = 1$ (RHS).

Proposition 5.1 (Robust Worst-Case Solution for at Most One Crash). *The solution to the problem*

$$\text{maximize} \inf_{\ell\in[0,1]} q(\beta,\ell) \text{ over all } \beta \in [0,1]$$

is given by the unique intersection point $\beta^\star$ from equation (5.4). *The corresponding strategy $\pi^{(\beta^\star)}$ solves the robust worst-case portfolio problem* (P).

Figure 4 illustrates the least favorable performance $q(\beta,\ell)$ as a function of the true crash size ℓ.

6. Multiple Crash Scenarios

In this section we extend the preceding analysis to financial markets with a known number $k \geq 1$ of crashes and an unknown crash size at each crash.

Efficiency for Multiple Crashes. We assume that each crash has its individual maximal size and there is no prior information about these sizes. In particular, the investor cannot learn from past crashes, and updating his crash size beliefs does not improve his performance. Thus he fixes his assumption on the maximal crash sizes in advance. In this setting, we define an efficiency criterion in analogy to (5.1).

We denote by $\pi^{(k,\ell)}$ the worst-case optimal investment strategy in an ideal model with a known number of at most k crashes of at most crash size ℓ. $\pi^{(k,\ell)}$ can be determined recursively via a series of ordinary differential equations (compare Korn and Steffensen (2007) or Seifried (2010)). Starting from $\pi^{(0)}(t) \triangleq \pi^M = \frac{\lambda}{\gamma\sigma^2}$, $t \in [0,T]$ we have

$$\begin{aligned}\frac{\mathrm{d}\pi^{(k,\ell)}(t)}{\mathrm{d}t} = \frac{1-\pi^{(k,\ell)}(t)\ell}{\ell}\Big\{&\lambda\left[\pi^{(k,\ell)}(t) - \pi^{(k-1,\ell)}(t)\right] \\ &- \tfrac{1}{2}\gamma\sigma^2\left[\pi^{(k,\ell)}(t)^2 - \pi^{(k-1,\ell)}(t)^2\right]\Big\}, \quad \pi^{(k,\ell)}(T) = 0\end{aligned} \tag{6.1}$$

for $k \geq 1$. As in the case $k = 1$, the worst-case optimal strategy $\pi^{(k,\ell)}$ is an indifference strategy.

Next, we determine the efficiency of the strategy $\pi^{(k,\ell)}$. We write $w(\beta,\ell;k) \triangleq w^{(k,\ell)}(x,\pi^{(k,\beta)})$ for the performance, in dollar values, attained by $\pi^{(k,\beta)}$ for the worst-case crash time scenario and least favorable crash size ℓ in a market with k possible crashes. As in Section 3 the associated efficiency is defined as the percentage of the optimal dollar performance attained with misspecified model parameters,

$$q(\beta,\ell;k) \triangleq w(\beta,\ell;k) \,/\, w(\ell,\ell;k).$$

Abbreviating $L_0(\beta,\ell;k) = \sum_{i=1}^{k} \ln(1-\pi^{(i,\beta)}(0)\ell)$, similarly as in Sections 4 and 5, we find

$$q(\beta,\ell;k) = \begin{cases} \int_0^T \left[\pi^{(k,\beta)}(t)\lambda - \frac{1}{2}\gamma\pi^{(k,\beta)}(t)^2\sigma^2\right]\mathrm{d}t - L_0(\ell,\ell;k), & \ell < \beta \\ L_0(\beta,\ell;k) - L_0(\ell,\ell;k) + \frac{1}{2}\frac{\lambda^2}{\gamma\sigma^2}T, & \ell \geq \beta. \end{cases} \tag{6.2}$$

Robust Worst-Case Optimal Strategy for $k > 1$. Arguing as in Section 5, we can invoke Lemma A.9 to show that the minimum of the function $\ell \mapsto q(\beta,\ell;k)$ is located at $\ell = 0$ or at $\ell = 1$, giving functions $\mu_0(\beta,k)$ and $\mu_1(\ell,k)$, respectively. The analysis of the two local minima proceeds along the same lines. As in Section 5 there is a unique

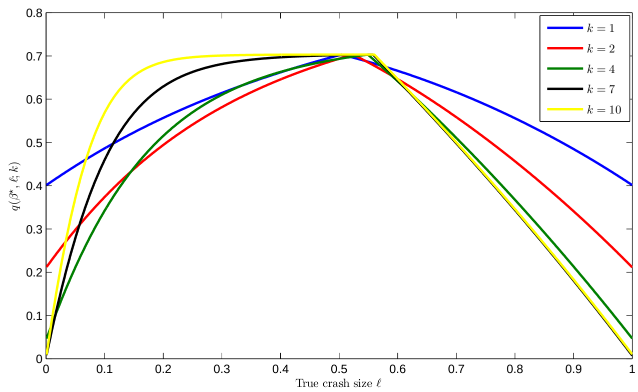

FIGURE 5. Least favorable performance $q(\beta^\star, \ell; k)$ as a function of ℓ (market parameters: $\mu = 0.20$, $\sigma = 0.40$, $r = 0.05$, $T = 10$) for $k = 1, 2, 4, 7, 10$ and $\gamma = 1$. The associated robustly optimal $\beta^\star(k)$ is located at the maxima of the respective curves.

intersection point $\beta^\star \in [0, 1]$ such that

$$\mu_0(\beta^\star, k) = \mu_1(\beta^\star, k) \tag{6.3}$$

where

$$\mu_0(\beta, k) = \int_0^T \left[\pi^{(k,\beta)}(t)\lambda - \tfrac{1}{2}\gamma\pi^{(k,\beta)}(t)^2\sigma^2\right] \mathrm{d}t$$
$$\mu_1(\beta, k) = L_0(\beta, 1; k) - L_0(1, 1; k) + \tfrac{1}{2}\tfrac{\lambda^2}{\gamma\sigma^2}T.$$

Again, as a consequence of Corollary A.7 and Lemma A.9 we obtain

Proposition 6.1 (Robust Worst-Case Solution for Multiple Crashes)**.** *The solution to the robust optimization problem*

$$\text{maximize} \inf_{\ell \in [0,1]} q(\beta, \ell; k) \text{ over all } \beta \in [0, 1]$$

is given by the unique intersection point $\beta^\star(k)$ from equation (6.3)*. The corresponding strategy $\pi^{(k,\beta^\star(k))}$ solves the robust worst-case portfolio problem* (P)*.*

Figure 5 illustrates the worst-case performance $q(\beta^\star, \ell; k)$ as a function of the true maximal crash size ℓ with the corresponding robustly optimal $\beta^\star(k)$ for $k = 1, 2, 4, 7, 10$. The value $\beta^\star(k) \approx 0.5$ in this figure is not representative; in general, $\beta^\star(k)$ depends on the risk aversion γ, compare Table 1. The figure also indicates that the robustly optimal $\beta^\star(k)$ increases with the number of maximal crashes k.

Large Number of Crashes. We now investigate the limiting case of a large number of crashes. The behavior of $\beta^\star(k)$ for large k large is illustrated in Figure 6. For all risk aversion parameters considered, $\beta^\star(k) = \beta^\star_\gamma(k)$ stabilizes quickly at a non-trivial stationary value $\hat{\beta}_\gamma$.

γ	0.25	0.50	0.75	1.00	2.00	3.00
$\hat{\beta}_\gamma$	0.86	0.64	0.58	0.55	0.52	0.50

TABLE 1. $\beta^\star_\gamma = \lim_{k\to\infty} \beta^\star_\gamma(k)$ for alternative risk aversion parameters γ.

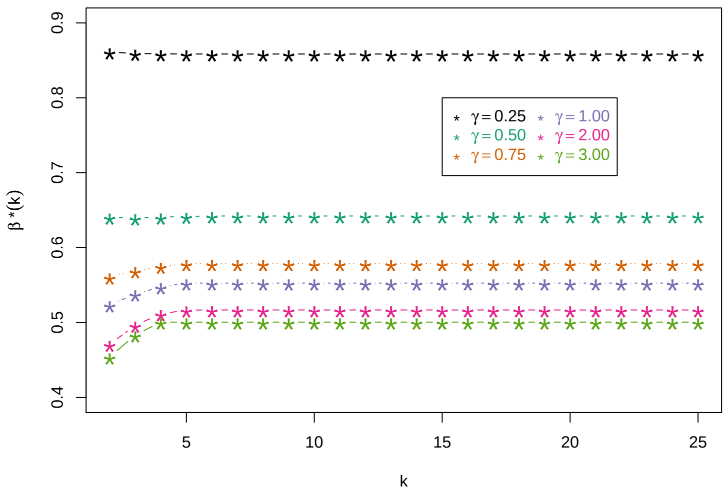

FIGURE 6. Robustly optimal $\beta^\star(k)$ as a function of the maximal number of crashes k.

The limiting values $\hat{\beta}_\gamma$, obtained by extrapolation, for alternative levels of relative risk aversion are displayed in Table 1.

We next consider the optimal value of the maximin criterion, i.e., the optimal efficiency

$$\mu(\beta^\star(k)) \triangleq \mu_0(\beta^\star(k), k) = \mu_1(\beta^\star(k), k).$$

Figure 7 confirms that the criterion $\mu(\beta^\star(k))$ is a decreasing function of the maximum number of crashes k and tends to 0 as $k \to \infty$. We may gain deeper insights by passing to a logarithmic scale: In fact, after a non-informative kick-in phase the average logarithmic contribution of an additional crash to the total efficiency quickly approaches a linear function. To examine this rigorously, for a maximal number of crashes $k \in \{10, 11, \dots, 25\}$ we have fitted a least squares regression line with intercept $\ln(\hat{A})$ and slope $\ln(\hat{b})$ to the log-linear model

$$\left(\ln \mu(\beta^\star(k))\right)/k = \ln(A) + \ln(b)\ln(k) + \text{noise}.$$

Our results for alternative specifications of the risk aversion parameter γ are summarized in Table 2. We obtain extraordinarily high values of R^2_{adj}, the fraction of variance explained by the model. These results imply that the $(k+1)^{\text{th}}$ crash decreases the optimal efficiency by a factor

γ	0.25	0.50	0.75	1.00	2.00	3.00
$\hat{A}$	1.599	1.622	1.568	1.505	1.330	1.230
$\hat{b}$	0.476	0.512	0.531	0.543	0.571	0.587
R^2_{adj}	0.998	0.998	0.998	0.997	0.995	0.994

TABLE 2. Fitted least squares coefficients and adjusted R^2 for different levels of risk aversion γ.

γ	0.25	0.50	0.75	1.00	2.00	3.00
$\hat{A}\hat{b}^{\ln 5}$	0.485	0.553	0.566	0.563	0.539	0.522
$\hat{A}\hat{b}^{\ln 10}$	0.290	0.348	0.365	0.369	0.365	0.361
$\hat{A}\hat{b}^{\ln 50}$	0.088	0.119	0.132	0.138	0.148	0.153
$\hat{A}\hat{b}^{\ln 100}$	0.053	0.075	0.085	0.090	0.100	0.106

TABLE 3. Impact of an additional crash (total number of crashes $k = 5, 10, 50, 100$) on optimal efficiency for different levels of risk aversion γ.

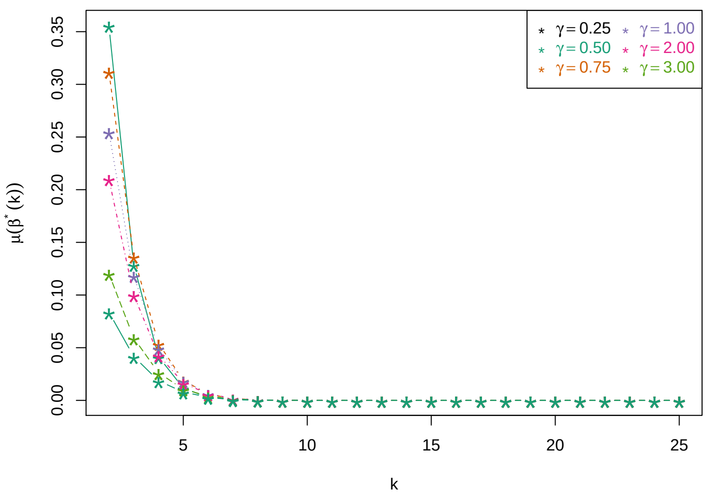

FIGURE 7. Optimal efficiency $\mu(\beta^\star(k))$ as a function of the number of crashes k.

$Ab^{\ln(k)}$. Considering $\ln(k)$ as (almost) constant for the relevant number of crashes, this means that the impact of an additional crash also stabilizes on a non-trivial level above 0. We report our results in Table 3 for alternative specifications of $k = 5, 10, 50, 100$. The corresponding least squares fits are illustrated in Figure 8: On an aggregated basis with a logarithmic y-axis we display the optimal criterion values $\mu(\beta^\star(k))$ together with the transformed fitted regression lines, confirming the excellent quality of the least squares fit.

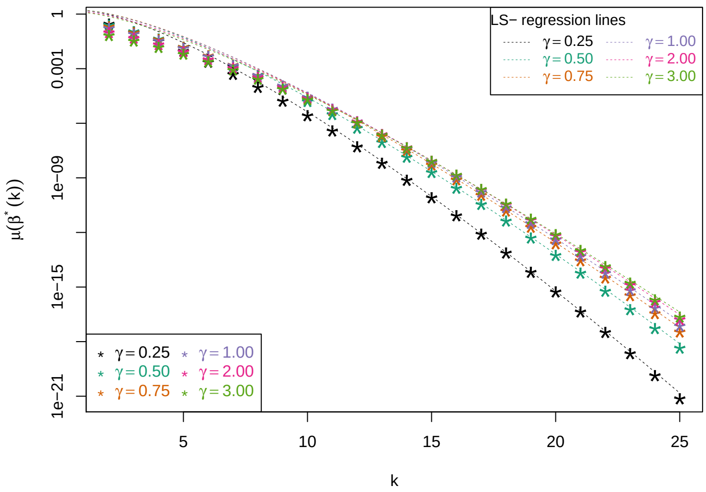

FIGURE 8. Optimal efficiency $\mu(\beta^\star(k))$ together with the transformed fitted least squares lines (fitted for $k \geq 10$) with a logarithmic y-axis.

7. Extension to Arbitrary Strategies

In this section we demonstrate that the robustly optimal worst-case crash strategy $\pi^{(k,\beta^*)}$ is in fact robustly optimal in the class $\mathcal{A}^{(k)}$ of all admissible strategies. More precisely, we show that the efficiency of any admissible strategy is dominated by that of a suitably chosen worst-case crash strategy. Recall from (3.6) and (P) that, explicating the θ– and θ'–optimal strategies $\pi^{(\theta)}$, $\pi^{(\theta')}$, our efficiency criterion for a guessed parameter θ' is given by

$$\inf_{\theta \in \Theta} w(\pi^{(\theta')}, \theta) \,/\, w(\pi^{(\theta)}, \theta). \tag{7.1}$$

In Propositions 5.1 and 6.1 we have established *parametric optimality* of $\pi^{\theta^\star}$, where $\theta^\star = (k, \beta^\star)$ denotes the maximizer in (7.1): $\pi^{\theta^\star}$ is optimal among all worst-case crash strategies π^θ where $\theta \in \Theta$. The set of those strategies is denoted by

$$\Pi^{(k,[0,1])} \triangleq \{\pi^{(k,\beta)} \,:\, \beta \in [0,1]\}.$$

We now demonstrate *global optimality* of this strategy in $\mathcal{A}^{(k)}$. Let

$$\bar{q}^{(k,\ell)}(\pi) \triangleq w^{(k,\ell)}(\pi) \,/\, w^{(k,\ell)}(\pi^{(k,\ell)}) \tag{7.2}$$

with $\pi \in \mathcal{A}^{(k)}$ an arbitrary strategy.

Theorem 7.1 (Completeness of Strategies)**.** *The set of strategies* $\Pi^{(k,[0,1])}$ *is complete in the sense that for any strategy* $\pi \in \mathcal{A}^{(k)}$ *there exists a* *strategy* $\pi^{(k,\beta)} \in \Pi^{(k,[0,1])}$ *(depending on* π*) such that*

$$\inf_{\ell} \bar{q}^{(k,\ell)}(\pi) \leq \inf_{\ell} \bar{q}^{(k,\ell)}(\pi^{(k,\beta)}).$$

In particular, $\pi^{\theta^\star}$ *is globally optimal for the robust worst-case portfolio problem* (P)*.*

The proof of Theorem 7.1 is delegated to Appendix A.

8. Conclusion and Outlook

Two of the most critical parameters in real-world applications of the worst-case approach to portfolio optimization are the total number of possible crashes and the maximum crash size. Both parameters play a crucial role in the determination of the worst-case optimal crash strategy. However, none of them is easily determined in practice. To address this, in this paper we have provided a robust formulation of the portfolio problem for worst-case crash scenarios with uncertain parameters. This formulation has led to very plausible, by no means overly pessimistic, robustly optimal crash sizes. Moreover, for a growing number of crashes they quickly converge to a stable limit.

Some further aspects which could make the worst-case approach even more realistic are left for future research. The most important one undoubtedly concerns the number of crashes. Promising results for growing number of crashes have been obtained in Section 6, indicating that an analysis of the limiting case $k \to \infty$ could be interesting. As the minimax efficiency for fixed time horizon T decreases (by a slowly increasing factor) for each new crash, an additional compensator needs to be introduced to avoid degeneracy. One idea could be to study a growing time horizon $T(k)$, or, similarly, to consider average crash frequencies k/T as an alternative parametrization of the problem.

Further extensions include the robustification with respect to other parameters, such as the excess return (as in Section 3), the time horizon, and post-crash market coefficients, as well as alternative utility functions.

Appendix A. Proofs

First, we study monotonicity of the worst-case optimal crash strategies with respect to the maximum crash size. Let us recall the following result.

Lemma A.1. *Let $I \subseteq \mathbb{R}$ be an interval and consider the parameterized family of ODEs on $[0,T]$*

$$y_\beta(0) = 0, \quad \dot{y}_\beta(t) = f(t, y_\beta, \beta).$$

Suppose that for each $\beta \in I$ there is a unique solution g_β on $[0,T]$ and assume that f is differentiable with respect to (y,β) for each $t \in [0,T]$. Moreover suppose that for each $t \in [0,T]$

$$\partial/\partial\beta\, f(t,y,\beta) < 0.$$

Then for each $t \in (0,T]$ the function $\beta \mapsto g_\beta(t)$ is strictly increasing.

Proof. Let $z_\beta(t) \triangleq \partial/\partial\beta\, y_\beta$. Then $\partial/\partial\beta\, g_\beta(t)$ is the unique solution to the linear inhomogeneous ODE

$$z_\beta(0) = 0, \quad \dot{z}_\beta(t) = a(t) + b(t)z_\beta(t)$$

with $a(t) \triangleq \partial/\partial\beta\, f(t,y,\beta)$ and $b(t) \triangleq \partial/\partial y\, f(t,y,\beta)$. The unique solution is given by

$$z_\beta(t) = \textstyle\int_0^t a(s) \exp\{\int_s^t b(u)\, du\}\, \mathrm{d}s$$

where the integrand is strictly negative. □

We now establish the desired monotonicity property of worst-case optimal strategies with the help of Lemma A.1.

Corollary A.2. *For each $t \in [0,T]$ and $k \geq 1$, the mapping $\beta \mapsto \pi^{(k,\beta)}(t)$ is strictly decreasing.*

Proof. We have $\pi^{0,\beta}(t) = \pi^M$ by Merton's result, and, since $\pi^{(k,\beta)}(t)$ is more conservative than $\pi^{((k-1),\beta)}(t)$,

$$\pi^{((k-1),\beta)}(t) > \pi^{(k,\beta)}(t) \text{ for each } t \in [0,T]. \tag{A.1}$$

By time-inversion and after rearranging terms, the ODE (6.1) for $\pi^{(k)}$ with terminal condition $\pi^{(k)}(T) = 0$ becomes

$$\dot{y}_k(t) = f_k(t, y_k(t), \beta), \quad y_k(0) = 0$$

where

$$\begin{aligned} f_k(t,y,\beta) &\triangleq -(\tfrac{1}{\beta} - y)\, \tfrac{\gamma\sigma^2}{2}\, h_k(t,y,\beta) \quad \text{with} \\ h_k(t,y,\beta) &\triangleq (y - \pi^{((k-1),\beta)}(t))[2\pi^M - y - \pi^{((k-1),\beta)}(t)]. \end{aligned} \tag{A.2}$$

It is clear that f_k is differentiable with respect to (y,β), where $\beta \in (0,1]$ and $y \in \mathbb{R}$, for every $t \in [0,T]$. Hence

$$\tfrac{2}{\gamma\sigma^2} \frac{\partial}{\partial\beta} f_k(t,y,\beta) = \tfrac{1}{\beta^2} h_k(t,y,\beta) - (\tfrac{1}{\beta} - y) \frac{\partial}{\partial\beta} h_k(t,y,\beta) \tag{A.3}$$

where

$$\frac{\partial}{\partial\beta} h_k(t,y,\beta) = -2 \frac{\partial}{\partial\beta} \pi^{((k-1),\beta)}(t)) \left\{ \pi^M - \pi^{((k-1),\beta)}(t) \right\}.$$

According to our introductory remark and (A.1), the first summand of $\partial/\partial\beta\, f_k(t,y,\beta)$ in (A.3) is always negative for $y = \pi^{(k,\beta)}(t)$, and so is the second one: The factor $1/\beta - y$ is positive for $y = \pi^{(k,\beta)}(t)$, and for $\partial/\partial\beta\, h_k(t,y,\beta)$, we argue by induction on the number of crashes. Let $k = 1$. Then $h_1(t,y,\beta)$ is constant in β, so $\partial/\partial\beta\, f_1(t,y,\beta)$ is negative, and hence by Lemma A.1 so is $\partial/\partial\beta\, \pi^{(1,\beta)}(t)$. Assume we have already shown negativity of $\partial/\partial\beta\, \pi^{(i,\beta)}(t)$ for $i = 1,\dots,k-1$. Then by induction, $\partial/\partial\beta\, h_k(t,y,\beta) \geq 0$, hence $\partial/\partial\beta\, f_k(t,y,\beta)$ is negative, and, again by Lemma A.1 so is $\partial/\partial\beta\, \pi^{(k,\beta)}(t)$. □

From the monotonicity we can immediately deduce the following:

Corollary A.3. *For each $k > 0$, the set of strategies $\Pi^{(k,[0,1])} \triangleq \{\pi^{(k,\beta)} : \beta \in [0,1]\}$ for $t \in [0,T]$ is bracketed by the constant strategies "Merton" and "Strictly Bond".*

Proof. $\pi \geq 0$ by definition and $\pi^{(k,0)} = \pi^M$; the rest is a consequence of Corollary A.2. □

Lemma A.4. *Any admissible strategy $\pi \in \mathcal{A}^{(k)}$ with $\pi > \pi^M$ on a time-set of positive Lebesgue measure is dominated by the Merton strategy.*

Proof. In the ideal world ($\ell = 0$) Merton is optimal, hence you can improve the strategy π setting it to Merton on $\pi > \pi^M$. For any $\ell > 0$, in case of a crash, with π you will lose more than Merton, and afterwards, you cannot beat Merton; see also Seifried (2010, Prop. 5.1). □

Lemma A.5. *On $(0,1]$, the mapping $\beta \mapsto \pi^{(k,\beta)}(t)$ is continuous in sup-norm on $[0,T]$, i.e.,*

$$F:\ (0,1] \to \mathcal{C}([0,1],\sup), \quad \beta \mapsto F(\beta) = \pi^{(k,\beta)}(t).$$

Moreover for each $t \in [0,T)$ we have $\lim_{\beta\downarrow 0} \pi^{(k,\beta)}(t) = \pi^{(k,0)}(t) = \pi^M$.

Proof. By Corollary A.3, we can bracket the range $\{\pi^{(k,\beta)}(t) : \beta \in [0,1], t \in [0,T]\}$ by $[0,\pi^M]$. Fix any $\beta_0 > 0$. Then on $[\beta_0,1] \times [0,\pi^M]$ the function f_k in (A.2) is Lipschitz continuous in the sup-norm of the range with a finite global Lipschitz constant. Now use Gronwall's Lemma to conclude that this is also true for the solution of the ODE. Fix $t \in [0,T)$ and let $\beta \downarrow 0$ (strictly). By monotonicity shown in Corollary A.2, $\beta \mapsto \pi^{(k,\beta)}(t)$ is strictly decreasing and hence by bracketing converges for $\beta \to 0$. But then $\beta \mapsto \partial/\partial\beta\, \pi^{(k,\beta)}(t)$ must converge to 0, which according to Lemma A.1 can only happen if $\partial/\partial\beta\, f_k(t,y,\beta)$ converges to 0. Now to this end by (A.2) and (A.3), necessarily $\pi^{(k,\beta)}(t) \to \pi^M$. □

Lemma A.6. *The function $w^{(k,\ell)}$ given in (4.2), (4.3), and (5.1) (for $k = 1$) and in (6.2) (for $k \geq 1$), understood as a mapping $\pi \in \mathcal{A}^{(k)} \mapsto w^{(k,\ell)}(\pi)$, is continuous in sup-norm.*

Proof. Immediate from (4.2), (4.3), (5.1), and (6.2)—evaluation at a point (i.e., $t = 0$) is continuous in sup-norm, and for the integral in (4.2) this follows from dominated convergence. □

Corollary A.7. *For $\ell \in \{0,1\}$, the mapping $\beta \mapsto w^{(k,\ell)}(\pi^{(k,\beta)})$ is continuous onto the range spanned by $w^{(k,\ell)}(\pi^{(k,1)})$, $w^{(k,\ell)}(\pi^{(k,0)})$. The same holds for $\beta \mapsto \bar{q}^{(k,\ell)}(\pi^{(k,\beta)})$ from (7.2).*

Proof. Continuity is the composition of Lemmata A.5 and A.6, which also implies that the image of $[0,1]$ under this map is an interval by the intermediate value theorem. Look at $\ell = 0$. Here $\pi^{(k,0)}$ is optimal, while by Corollary A.2, $\pi^{(k,1)}$ is the pointwise minimum of $\beta \mapsto \pi^{(k,\beta)}$. Hence, it is pointwise furthest away from Merton and thus $w^{(k,0)}(\pi^{(k,1)}) = \min_\beta w^{(k,0)}(\pi^{(k,\beta)})$. The case $\ell = 1$ is similar. In both cases the set

$$I_\ell^{(k)} \triangleq \{w^{(k,\ell)}(\pi^{(k,\beta)}) \,:\, \beta \in [0,1]\}$$

is bracketed by $\{w^{(k,\ell)}(\pi^{(k,\beta)}) \,:\, \beta \in \{0,1\}\}$. For fixed ℓ, the assertion for $\bar{q}^{(k,\ell)}$ is just a restandardization. □

Proposition A.8. *For each $\beta \in [0,1]$, the worst-case optimal strategy $\pi^{(k,\beta)}$ also solves the constrained optimization problem to*

$$\text{maximize } w^{(k,0)}(\pi) \text{ over } \pi \in \mathcal{A}^{(k)} \text{ subject to } w^{(k,1)}(\pi) \geq w^{(k,1)}(\pi^{(k,\beta)})$$

as well as the constrained optimization problem to

$$\text{maximize } w^{(k,1)}(\pi) \text{ over } \pi \in \mathcal{A}^{(k)} \text{ subject to } w^{(k,0)}(\pi) \geq w^{(k,0)}(\pi^{(k,\beta)}).$$

The same also holds with w replaced by $\bar{q}$.

Proof. Start with the first assertion on w. Suppose that the strategy $\pi^{(k,\beta)}$ is suboptimal in the restricted problem. Then there is some $\pi_0 \in \mathcal{A}^{(k)}$ with $w^{(k,1)}(\pi_0) \geq w^{(k,1)}(\pi^{(k,\beta)})$ and $w^{(k,0)}(\pi_0) > w^{(k,0)}(\pi^{(k,\beta)})$, which is true only if we are riskier than $\pi^{(k,\beta)}$. This means that the worst case for π_0 is that "exactly k crashes happen". But by worst-case optimality of $\pi^{(k,\beta)}$, $w^{(k,\beta)}(\pi_0) < w^{(k,\beta)}(\pi^{(k,\beta)})$, or

$$w^{(k,\beta,\text{exactly k crashes happen})}(\pi_0) < \ln(1 - \pi^{(k,\beta)}(0)\beta). \tag{A.4}$$

Now the strategy π_0, after the k^{th} crash has happened, cannot beat $\pi^{(k,\beta)}$, which uses Merton afterwards. So assume without loss that π_0 also uses Merton afterwards. But (A.4) implies that at crash time t, $\pi_0(t) > \pi^{(k,\beta)}(t)$, and hence is even more affected when instead of β, the crash size is 1. Hence $w^{(k,1)}(\pi_0) < w^{(k,1)}(\pi^{(k,\beta)})$, which is a contradiction.

For the second assertion suppose that there is some $\pi_1 \in \mathcal{A}^{(k)}$ with $w^{(k,0)}(\pi_1) \geq w^{(k,0)}(\pi^{(k,\beta)})$ and $w^{(k,1)}(\pi_1) > w^{(k,1)}(\pi^{(k,\beta)})$, which is true only if π_1 has stronger crash protection than $\pi^{(k,\beta)}$. This means that the worst case for π_1 is "no crash". But by worst-case optimality of $\pi^{(k,\beta)}$, this gives the contradiction

$$w^{(k,0)}(\pi_1) = w^{(k,\beta)}(\pi_1) < w^{(k,\beta)}(\pi^{(k,\beta)}) = w^{(k,1)}(\pi^{(k,\beta)}).$$

As above, the assertion concerning $\bar{q}$ is merely a restandardization. □

Lemma A.9. *For each $\beta \in (0,1]$, the mapping $\ell \mapsto \bar{q}^{(k,\ell)}(\pi^{(k,\beta)})$ is decreasing for $\ell > \beta$, while it is increasing for $\ell < \beta$. In particular,*

$$\inf_\ell \bar{q}^{(k,\ell)}(\pi^{(k,\beta)}) = \min\left(\bar{q}^{(k,0)}(\pi^{(k,\beta)}), \bar{q}^{(k,1)}(\pi^{(k,\beta)})\right).$$

Proof. Look at (5.1) respectively (6.2) and set $\pi_\ell^{(k)} \triangleq \pi^{(k,\ell)}(0)$. For $\ell < \beta$, we have to consider $\partial/\partial\ell \sum_{i=1}^k \{-\ln(1-\pi_\ell^{(i)}\ell)\}$. Now the optimal worst-case strategy $\pi^{(k,\ell)}$ is indifferent between one more or no more crash, hence solves ODE (6.1), which after integration can be written as

$$2(\ln(1-\pi_\ell^{(i)}\ell))/(\sigma^2\gamma) =$$
$$\textstyle\int_0^T [(\pi^{(i,\ell)}(s) - \pi^{(i-1,\ell)}(s))][2\pi^M - \pi^{(i,\ell)}(s) - \pi^{(i-1,\ell)}(s)]\,\mathrm{d}s.$$

Since the integrand is continuously differentiable on $[0,T]$, we may interchange integration and differentiation and get

$$\tfrac{1}{\sigma^2\gamma}\frac{\partial}{\partial\ell}\ln(1-\pi_\ell^{(i)}\ell) = \textstyle\int_0^T \rho_i(s) - \rho_{i-1}(s)\,\mathrm{d}s \tag{A.5}$$

for $\rho_i(s) = (\pi^M - \pi^{(i,\ell)}(s))\,\partial/\partial\ell\,\pi^{(i,\ell)}(s)$. Summing up over i, equation (A.5) is telescoping,

$$\frac{\partial}{\partial\ell}\sum_{i=1}^k \left\{-\ln(1-\pi_\ell^{(i)}\ell)\right\} = -\sigma^2\gamma\textstyle\int_0^T \rho_k(s)\,\mathrm{d}s$$

and as ρ_k is negative, the assertion follows.

For $\ell > \beta$, we have to consider

$$\frac{\partial}{\partial\ell}\sum_{i=1}^k \ln\frac{1-\pi_\beta^{(i)}\ell}{1-\pi_\ell^{(i)}\ell} = \sum_{i=1}^k h(\pi_\ell^{(i)},\ell) - h(\pi_\beta^{(i)},\ell) + \frac{\ell\frac{\partial}{\partial\ell}\pi_\ell^{(i)}}{1-\pi_\ell^{(i)}\ell}$$

for $h(x,a) = x/(1-ax)$. Now, as $h(x,a)$ for $ax < 1$ is increasing in x, the sum of the first two summands in the outer sum is negative, while the last summand by Corollary A.2 is negative anyway. $\square$

With these preparations we can now give the

Proof of Theorem 7.1. Fix any $\pi \in \mathcal{A}^{(k)}$. By Lemma A.4, we may assume $\pi \le \pi^M$. Because of Lemma A.9, it suffices to look at $\ell \in \{0,1\}$ (the worst case for π is at most smaller). Define $q_j = \bar{q}^{(k,j)}(\pi)$, $j = 0,1$, where we may exclude the case

$$q_0 < \bar{q}^{(k,0)}(\pi^{(k,1)})(< \bar{q}^{(k,1)}(\pi^{(k,1)}) = 1).$$

Now for $\check{q} \triangleq \min(q_0,q_1)$ consider the cases $\check{q} = q_0$ and $\check{q} = q_1$ separately. Then after our exclusion of suboptimal π's outside $I_\ell^{(k)}$, in each case, $\check{q} = q_j$, $j = 0,1$, Corollary A.7 gives us a β_j, $j = 0,1$ such that $\bar{q}^{(k,j)}(\pi^{(k,\beta_j)}) = q_j$. By Proposition A.8, in the opposite situation $\ell = 1-j$, the corresponding strategy $\pi^{(k,\beta_j)}$ is optimal on the whole set of strategies $\mathcal{A}^{(k)}$ subject to $\bar{q}^{(k,j)}(\pi^{(k,\beta_j)}) \ge q_j$. Hence $\bar{q}^{(k,1-j)}(\pi^{(k,\beta_j)}) \ge \bar{q}^{(k,1-j)}(\pi)$. $\square$

Appendix B. Illustration of the rmx Approach in Robust Statistics

Generally, the rmx approach in robust statistics leads to very reasonable, by no means overly pessimistic procedures. In particular it compares very well with other approaches from robust statistics to select this radius. In the two most prominent ones one either selects the radius maximal, just looking at stability only, which gives the *most bias robust estimator* MBRE. Or, in an approach due to Anscombe (1960), one fixes an insurance premium in terms of ARE in the ideal model, which is paid for outlier protection—the standard default in the community is 95% leading to the 95%-efficiency-tuned *optimally bias robust estimator* $\mathrm{OBRE}_{95\%}$.

We illustrate this in a set of parametric models, i.e., the Gaussian location model $\mathcal{N}(\mu, 1)$, $\mu \in \mathbb{R}$, the Gaussian scale model $\mathcal{N}(0, \sigma)$, $\sigma > 0$, the Gaussian location-scale model $\mathcal{N}(\mu, \sigma)$, $\theta = (\mu, \sigma) \in \mathbb{R} \times \mathbb{R}_{>0}$, the Poisson model $\mathrm{Pois}(\lambda)$, $\lambda > 0$, at $\lambda = 1$, and the Generalized Pareto shape-scale model $\mathrm{GPD}(0, \xi, \beta)$, $\theta = (\xi, \beta) \in \mathbb{R}^2_{>0}$, at $\xi = 0.7$. These models are surrounded by respective ε-contamination neighborhoods of unknown radius. The respective least favorable AREs are summarized in Table 4, the entries of which can easily be reproduced in `R` (see R Core Team (2013)) with script `AnscombeOrNot.R` in CRAN-pkg `ROptEst` (see Kohl and Ruckdeschel (2013)). Note that in this context, in the Gaussian location-scale model the well-known robust estimator consisting in median and median of absolute deviations only achieves a least favorable efficiency of 51% (compared to 76% of the RMXE). In addition, for the rmx procedure, we also list the least favorable number of outliers #out in 100 observations in each situation, underlining that the least favorable situation is by no means overly pessimistic.

model\estim.	MLE	$\mathrm{OBRE}_{95\%}$	MBRE	RMXE; [#out]
$\mathcal{N}(\mu, 1)$	0%	60%	64%	**85%**; [6]
$\mathcal{N}(0, \sigma)$	0%	19%	37%	**67%**; [5]
$\mathcal{N}(\mu, \sigma)$	0%	33%	57%	**76%**; [6]
$\mathrm{Pois}(\lambda)$	0%	48%	82%	**86%**; [4]
$\mathrm{GPD}(0, \xi, \beta)$	0%	14%	44%	**68%**; [5]

Table 4. Least favorable efficiency $\sup_{r'} \inf_r q(r', r)$ of different procedures at different models (in the ε-contamination model).

References

Anscombe, F.J., 1960, Rejection of outliers, *Technometrics* 2, 123–147.

Beyer, H.-G., and B. Sendhoff, 2007, Robust optimization - a comprehensive survey, *Computer Methods in Applied Mechanics and Engineering* 196, 3190–3218.

Cont, R., 2006, Model uncertainty and its impact on the pricing of derivative instruments, *Mathematical Finance* 16, 519–547.

———, R. Deguest, and G. Scandolo, 2010, Robustness and sensitivity analysis of risk measurement procedures, *Quantitative Finance* 10, 593–606.

Föllmer, Hans, Alexander Schied, and Stefan Weber, 2009, Robust preferences and robust portfolio choice, *Mathematical Modelling and Numerical Methods in Finance (ed. P. Ciarlet, A. Bensoussan, Q. Zhang), Handbook of Numerical Analysis* 15, 29–88.

Gilboa, Itzhak, and David Schmeidler, 1989, Maxmin expected utility with non-unique prior, *Journal of Mathematical Economics* 18, 141–153.

Hampel, F. R., E. M. Ronchetti, P. J. Rousseeuw, and W. A. Stahel, 1986, *Robust Statistics: The Approach Based on Influence Functions* (John Wiley & Sons).

Hua, Philip, and Paul Wilmott, 1997, Crash courses, *Risk* 10, 64–67.

Knight, Frank H., 1921, *Risk, Uncertainty, and Profit* (Houghton Mifflin).

Kohl, M., and P. Ruckdeschel, 2013, *ROptEst: Optimally robust estimation.* R package, version 0.8, `http://cran.r-project.org`.

Korn, Ralf, and Olaf Menkens, 2005, Worst-case scenario portfolio optimization: a new stochastic control approach, *Mathematical Methods of Operations Research* 62, 123–140.

Korn, Ralf, and Mogens Steffensen, 2007, On worst-case portfolio optimization, *SIAM Journal on Control and Optimization* 46, 2013–2030.

Korn, Ralf, and Paul Wilmott, 2002, Optimal portfolios under the threat of a crash, *International Journal of Theoretical and Applied Finance* 5, 171–187.

Krätschmer, V., A. Schied, and H. Zähle, 2012, Comparative and qualitative robustness for law-invariant risk measures, *Working Paper.*

Merton, Robert M., 1969, Lifetime portfolio selection under uncertainty: The continuous-time case, *Review of Economics and Statistics* 51, 247–257.

——— , 1971, Optimum consumption and portfolio rules in a continuous-time model, *Journal of Economic Theory* 3, 373–413.

Pham, Huyên, 2009, *Continuous-Time Stochastic Control and Optimization with Financial Applications* (Springer).

R Core Team, 2013, *R: A Language and Environment for Statistical Computing*, R Foundation for Statistical Computing, Vienna (ISBN 3-900051-07-0).

Rieder, Helmut, 1994, *Robust Asymptotic Statistics* (Springer).

——— , Matthias Kohl, and Peter Ruckdeschel, 2008, The cost of not knowing the radius, *Statistical Methods and Applications* 17, 13–40.

Rogers, L.C.G., 2013, *Optimal Investment* (Springer).

Schied, Alexander, 2005, Optimal investment for robust utility functionals in complete markets, *Mathematics of Operations Research* 30, 750–764.

Seifried, Frank T., 2010, Optimal investment for worst-case crash scenarios: A martingale approach, *Mathematics of Operations Research* 35, 559–579.

Zähle, H., 2013, Qualitative robustness of statistical functionals under strong mixing, *Working Paper.*

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