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# Vorwort

Das Tätigkeitsfeld des Fraunhofer-Instituts für Techno- und Wirtschaftsmathematik ITWM umfasst anwendungsnahe Grundlagenforschung, angewandte Forschung sowie Beratung und kundenspezifische Lösungen auf allen Gebieten, die für Techno- und Wirtschaftsmathematik bedeutsam sind.

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Darüber hinaus bietet die Reihe ein Forum für die Berichterstattung über die zahlreichen Kooperationsprojekte des Instituts mit Partnern aus Industrie und Wirtschaft.

Berichterstattung heißt hier Dokumentation des Transfers aktueller Ergebnisse aus mathematischer Forschungs- und Entwicklungsarbeit in industrielle Anwendungen und Softwareprodukte – und umgekehrt, denn Probleme der Praxis generieren neue interessante mathematische Fragestellungen.



Prof. Dr. Dieter Prätzel-Wolters  
Institutsleiter

Kaiserslautern, im Juni 2001



# A DIRECTION SPLITTING APPROACH FOR INCOMPRESSIBLE BRINKMAN FLOW

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**ABSTRACT.** The direction splitting approach proposed earlier in [6], aiming at the efficient solution of Navier-Stokes equations, is extended and adopted here to solve the Navier-Stokes-Brinkman equations describing incompressible flows in plain and in porous media. The resulting pressure equation is a perturbation of the incompressibility constrained using a direction-wise factorized operator as proposed in [6]. We prove that this approach is unconditionally stable for the unsteady Navier-Stokes-Brinkman problem. We also provide numerical illustrations of the method's accuracy and efficiency.

## 1. INTRODUCTION

Flows in highly porous media occur often in industrial and scientific applications. Examples are the flows through various filters (air, oil, water filters, etc.) flow of helium in pebble-bed nuclear reactors, various physiological flows like the flow in the eye of glaucoma patients, flows in mangrove swamps etc. If the porosity of the media is high, such flows are usually modeled with the Brinkman equations. The Navier-Stokes-Brinkman model is positioned between the Darcy model for the flow in media with a very low porosity and the Navier-Stokes equations for flows with infinitely large porosity. Further on, prescribing properly the coefficients of the Navier-Stokes-Brinkman equations, one can

use it to describe the flow in plain (pure fluid) region with or without embedded obstacles, coupled to the flow in highly porous media. In all these cases, the flow field is a subject to the incompressibility constraint. The imposition of this constraint is a well-known hurdle for any discretization algorithm for such problems. In the case of unsteady flows described by pure Navier-Stokes equations, probably the most popular algorithms for the imposition of incompressibility are the so-called projection methods. These methods are pioneered by Chorin [4] and Temam [12]<sup>1</sup>. For a recent and comprehensive review on projection methods the reader is referred to [5]. All projection methods in fact are a discretization of a perturbation of the continuity equation which comes in the form of a pressure Poisson equation ( $L^2$  projection onto a divergence-free subspace of the velocity space). The solution of this Poisson equation can often require the major computational effort in an incompressible flow solver. This difficulty was recently circumvented in [6] where it was shown that the Laplace operator is one of many possible perturbation operators which would yield a stable and accurate incompressible algorithm for unsteady flows. Also there, a direction-wise factorized operator of the form  $\prod_{i=1}^d (I - \partial_{x_i x_i})$  (d being the dimension of the problem) was proposed instead of the classical Laplace operator. In the present article we extend this approach to the case of incompressible Brinkman flow and to the case of Navier-Stokes-Brinkman flow (the latter concerns coupled flow in plain and in porous media). We prove that if the momentum equation is not split direction-wise, the resulting algorithm is unconditionally stable. The direction splitting of the momentum equation, however, is more difficult than in the Navier-Stokes case because in general, the resulting one-dimensional operators do not commute unless the geometry of the entire domain and the geometry of the porous subdomain are simple rectangles/parallelepipeds.

## 2. THE FICTITIOUS DOMAIN BRINKMAN EQUATIONS

Consider the Brinkman equations in a domain  $\tilde{\Omega} \subset \mathbb{R}^d$  ( $d = 2, 3$ ) with a Lipschitz boundary  $\Gamma = \partial\tilde{\Omega}$ :

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<sup>1</sup>The authors have recently discovered that a similar velocity-pressure decoupling approach was proposed earlier in the famous article of Harlow and Welch [8] which also proposed the MAC staggered grid setting for the Stokes problem with free boundaries. Thus, some credit for pioneering the projection methods should be given to this article.

$$(1) \quad \begin{cases} \partial_t \tilde{\mathbf{u}} - \tilde{\nu} \Delta \tilde{\mathbf{u}} + \nabla p + \frac{\tilde{\nu}}{\tilde{k}} \tilde{\mathbf{u}} = \tilde{\mathbf{f}} & \text{in } \tilde{\Omega} \times [0, T], \\ \nabla \cdot \tilde{\mathbf{u}} = 0 & \text{in } \tilde{\Omega} \times [0, T], \\ \tilde{\mathbf{u}}|_{\partial\tilde{\Omega}} = 0 & \text{in } [0, T], \quad \text{and } \tilde{\mathbf{u}}|_{t=0} = \tilde{\mathbf{u}}_0 \text{ in } \tilde{\Omega}, \end{cases}$$

where  $\tilde{\nu}$  is the kinematic viscosity of the fluid and  $\tilde{k}$  is a permeability that can vary throughout  $\tilde{\Omega}$  representing the properties of the fluid and/or the porous media. Let  $\Omega$  be a rectangle in 2D and parallelepiped in 3D such that  $\tilde{\Omega} \subseteq \Omega$  and consider the following extension of the data

$$(2) \quad \nu = \tilde{\nu}, \quad \text{in } \Omega,$$

$$(3) \quad \mathbf{f} = \begin{cases} \tilde{\mathbf{f}}, & \text{in } \tilde{\Omega}, \\ 0, & \text{in } \Omega \setminus \tilde{\Omega}, \end{cases}$$

$$(4) \quad \mathbf{u}_0 = \begin{cases} \tilde{\mathbf{u}}_0, & \text{in } \tilde{\Omega}, \\ 0, & \text{in } \Omega \setminus \tilde{\Omega}, \end{cases}$$

$$(5) \quad k(\mathbf{x}) = \begin{cases} \tilde{k}, & \text{in } \tilde{\Omega}, \\ \nu\epsilon, & \text{in } \Omega \setminus \tilde{\Omega}, \end{cases}$$

where  $0 < \epsilon \ll 1$  is a penalty parameter used to enforce the boundary conditions on  $\partial\tilde{\Omega}$ . Then the  $L^2$ -penalty fictitious domain formulation of the problem in  $\Omega$  reads

$$(6) \quad \begin{aligned} \partial_t \mathbf{u}_\varepsilon - \nu \Delta \mathbf{u}_\varepsilon + \nabla p_\varepsilon + \frac{\nu}{k} \mathbf{u}_\varepsilon &= \mathbf{f}, \quad x \in \Omega \times [0, T] \\ \nabla \cdot \mathbf{u}_\varepsilon &= 0 \text{ in } \Omega \times [0, T], \\ \mathbf{u}_\varepsilon|_{\partial\Omega} &= 0 \text{ in } [0, T], \quad \text{and } \mathbf{u}_\varepsilon|_{t=0} = \mathbf{u}_0 \text{ in } \Omega. \end{aligned}$$

It is well known (see for example [2]) that the following result holds under sufficient regularity assumptions on the data and the domain:

$$u_\varepsilon(x) \xrightarrow[\varepsilon \rightarrow 0]{} u(x), \quad x \in \tilde{\Omega}.$$

The order of convergence depends on the regularity of the data and the domain, but it is at least  $O(\epsilon^{1/2})$ .

### 3. NUMERICAL ALGORITHM

**3.1. Fractional time step discretization.** It is not a new idea to use direction splitting techniques for the momentum equations in order to increase the computational efficiency. However, often the cost of computing the pressure, usually done via projection methods resulting in a Neumann problem for a Poisson equation, is dominant. Therefore,

it is attractive to use a simpler perturbation of the incompressibility constraint which allows for a more efficient pressure computation. We discretize equations (6) using the splitting procedure introduced in [6]. Since the permeability can vary greatly throughout the computational domain, the direction splitting of the momentum equation, in case of an implicit treatment of the Brinkman term  $\nu \mathbf{u}_\varepsilon / k$ , is not very straightforward. To understand the problem, let us consider the Douglas splitting for the first equation in (6) (see [9]) in the following factorized form

(7)

$$\prod_{i=1}^d \left( b(\mathbf{x})I - \frac{\nu \Delta t}{2b(\mathbf{x})} \partial_{x_i x_i} \right) \frac{\mathbf{u}^{n+1} - \mathbf{u}^n}{\Delta t} - \nu \Delta \mathbf{u}^n + \frac{\nu}{k} \mathbf{u}^n + \nabla p^{*,n+1/2} = \mathbf{f}^{n+1/2},$$

with  $b(\mathbf{x}) = (1 + \nu \Delta t / (2k))^{1/d}$  and  $I$  being the identity operator. It is clear that since  $b(\mathbf{x})^{-1} \partial_{x_1 x_1}$  and  $b(\mathbf{x})^{-1} \partial_{x_2 x_2}$  do not commute in general, the product operators of the type  $b(\mathbf{x})^{-1} \partial_{x_1 x_1} b(\mathbf{x})^{-1} \partial_{x_2 x_2}$  do not necessarily generate a positive and self-adjoint bilinear form which affects the stability of the scheme. Therefore, in this paper we do not consider the possibility to split direction-wise the momentum equation. As a result, the full 2D/3D momentum problem must be solved by means of an iterative procedure. The reader is referred to [3] for some possibilities for a direction splitting of the momentum equation in the non-commutative case. In the sequel of the paper we will omit the subscript  $\varepsilon$  of the solution to the penalized problem (6) and abusing somewhat the notation will denote the solution of the semi-discrete (in time only) splitting scheme by  $\mathbf{u}$  and  $p$ . Given some values  $\mathbf{u}^n, p^{n-1/2}, \phi^{n-1/2}$  the splitting scheme proceeds as follows:

### Pressure predictor:

$$(8) \quad p^{*,n+1/2} = p^{n-1/2} + \phi^{n-1/2}.$$

### Velocity update:

$$(9) \quad \begin{aligned} & \frac{\mathbf{u}^{n+1} - \mathbf{u}^n}{\Delta t} - \frac{1}{2} \nu \Delta (\mathbf{u}^{n+1} + \mathbf{u}^n) + \nabla p^{*,n+1/2} + \\ & \frac{\nu}{2k} (\mathbf{u}^{n+1} + \mathbf{u}^n) = \mathbf{f}^{n+1/2}, \quad \mathbf{u}^{n+1}|_{\partial\Omega} = 0. \end{aligned}$$

### Pressure-corrector:

$$(10) \quad A\phi^{n+1/2} = -\frac{1}{\Delta t} \nabla \cdot \mathbf{u}^{n+1},$$

where  $A = \prod_{i=1}^d (I - \partial_{x_i x_i})$  together with homogeneous Neumann boundary conditions on  $\partial\Omega$ . Note that this operator is factorized alongside the spatial directions and is therefore much easier to invert than the

usual for projection methods Laplace operator. It is also straightforward to prove (see [6]) that the bilinear form  $a(p, q) := \int_{\Omega} q A p d\mathbf{x}$  satisfies the following properties:

$$(11) \quad a \text{ is symmetric,} \quad \text{and} \quad \|\nabla q\|_{\mathbf{L}^2}^2 \leq a(q, q), \quad \forall q \in D(A).$$

where  $D(A)$  is the domain of  $A$ .

**Pressure update:**

$$(12) \quad p^{n+1/2} = p^{n-1/2} + \phi^{n+1/2} - \frac{1}{2} \chi \nu \nabla \cdot (\mathbf{u}^{n+1} + \mathbf{u}^n),$$

where  $\chi \in [0, 1]$ .

The stability of the scheme is guaranteed by the following theorem.

**Theorem 3.1.** *Assume that the solution to (6) is smooth enough. There exist  $c$ , independent of  $\Delta t$ , such that for all  $T$  and all  $\chi \in [0, 1]$  the solution to (8-12), satisfies the following stability estimate:*

$$(13) \quad \begin{aligned} & \|\mathbf{u}\|_{\ell^\infty(0,T;\mathbf{L}^2)}^2 + \frac{\Delta t}{2} \|\nu^{1/2} \nabla \mathbf{u}\|_{\ell^\infty(\Delta t,T;\mathbf{L}^2)}^2 + \Delta t^2 \|p\|_{\ell^\infty(\frac{\Delta t}{2},T-\frac{\Delta t}{2};D(A))}^2 + \\ & \frac{\Delta t}{2} \left\| \left( \frac{\nu}{k} \right)^{1/2} \mathbf{u} \right\|_{\ell^\infty(\Delta t,T;\mathbf{L}^2)}^2 \leq c \left( \|\mathbf{u}^0\|_{\mathbf{L}^2}^2 + \Delta t^2 \|p^{-1/2}\|_A^2 + \right. \\ & \left. \frac{\Delta t}{2} \|\nu^{1/2} \nabla \mathbf{u}^0\|_{\mathbf{L}^2}^2 + \frac{\Delta t}{2} \left\| \left( \frac{\nu}{k} \right)^{1/2} \mathbf{u}^0 \right\|_{\mathbf{L}^2}^2 + \nu^{-1} \|\mathbf{f}\|_{\ell^2(\frac{\Delta t}{2},T-\frac{\Delta t}{2};\mathbf{H}^{-1})}^2 \right). \end{aligned}$$

*Proof.* The proof follows along the same lines as the stability of the direction splitting scheme in case of the Navier-Stokes equations given in [6]. Nevertheless, there are some differences and we provide a brief sketch here for the case  $\chi = 0$  and  $\mathbf{f} = 0$  only. This stability estimate can then be used to study the accuracy of the scheme similarly to the case of the rotational pressure correction method studied in [7]. We first multiply (9) by  $2\Delta t \mathbf{u}^{n+1}$ , integrate over  $\Omega$ , and use the identity  $2(a-b, a) = \|a\|^2 + \|a-b\|^2 - \|b\|^2$  and the Young's inequality to obtain

$$(14) \quad \begin{aligned} & \|\mathbf{u}^{n+1}\|_{\mathbf{L}^2}^2 + \|\mathbf{u}^{n+1} - \mathbf{u}^n\|_{\mathbf{L}^2}^2 + \frac{\Delta t}{2} \|\nu^{1/2} \nabla \mathbf{u}^{n+1}\|_{\mathbf{L}^2}^2 \\ & + 2\Delta t (\nabla p^{*,n+1/2}, \mathbf{u}^{n+1}) + \frac{\Delta t}{2} \left\| \left( \frac{\nu}{k} \right)^{1/2} \mathbf{u}^{n+1} \right\|_{\mathbf{L}^2}^2 \leq \\ & \|\mathbf{u}^n\|_{\mathbf{L}^2}^2 + \frac{\Delta t}{2} \|\nu^{1/2} \nabla \mathbf{u}^n\|_{\mathbf{L}^2}^2 + \frac{\Delta t}{2} \left\| \left( \frac{\nu}{k} \right)^{1/2} \mathbf{u}^n \right\|_{\mathbf{L}^2}^2. \end{aligned}$$

Now we use the properties (11) of the operator  $A$  to deduce that the pressure correction  $(p^{n+1/2} - p^{n-1/2}) \in D(A)$  solves the following problem for  $n \geq 0$ :

$$(15) \quad a(p^{n+1/2} - p^{n-1/2}, q) = -\Delta t^{-1} (\nabla \cdot \mathbf{u}^{n+1}, q), \quad \forall q \in D(A).$$

Testing it with  $2\Delta t^2 p^{*,n+1/2} := 2\Delta t^2(2p^{n-1/2} - p^{n-\frac{3}{2}})$  (in case  $\chi = 0$ ) and using the symmetry and coercivity of  $a(\cdot, \cdot)$ , we obtain exactly as in the proof of Theorem 4.2 in [6], that

$$(16) \quad \begin{aligned} -2\Delta t(\nabla \cdot \mathbf{u}^{n+1}, p^{*,n+1/2}) &= \Delta t^2 (\|p^{n+1/2}\|_A^2 - \\ &\quad \|p^{n-1/2}\|_A^2 + \|\delta p^{n-1/2}\|_A^2 - \|\delta^2 p^{n+1/2}\|_A^2). \end{aligned}$$

where  $\delta p^{n-1/2} = p^{n-1/2} - p^{n-3/2}$  is the usual difference operator. Again as in the proof of Theorem 4.2 in [6], the control on  $\|\delta^2 p^{n+1/2}\|_A^2$  is obtained subtracting (15) at time  $t^n$  from (15) at time  $t^{n+1}$  and by testing the result with  $\Delta t \delta^2 p^{n+1/2}$ ,

$$\begin{aligned} \Delta t \|\delta^2 p^{n+1/2}\|_A^2 &= -(\nabla \cdot (\mathbf{u}^{n+1} - \mathbf{u}^n), \delta^2 p^{n+1/2}) = (\mathbf{u}^{n+1} - \mathbf{u}^n, \nabla \delta^2 p^{n+1/2}) \\ &\leq \|\mathbf{u}^{n+1} - \mathbf{u}^n\|_{\mathbf{L}^2} \|\nabla \delta^2 p^{n+1/2}\|_{\mathbf{L}^2}. \end{aligned}$$

Then the coercivity property of the bilinear form  $a$  implies that

$$\Delta t \|\nabla \delta^2 p^{n+1/2}\|_{\mathbf{L}^2} \|\delta^2 p^{n+1/2}\|_A \leq \|\mathbf{u}^{n+1} - \mathbf{u}^n\|_{\mathbf{L}^2} \|\nabla \delta^2 p^{n+1/2}\|_{\mathbf{L}^2},$$

which yields the inequality  $\Delta t^2 \|\delta^2 p^{n+1/2}\|_A^2 \leq \|\mathbf{u}^{n+1} - \mathbf{u}^n\|_{\mathbf{L}^2}^2$ . This bound together with (16) gives the following bound on the pressure gradient term in (14)

$$(17) \quad \begin{aligned} \Delta t^2 (\|p^{n+1/2}\|_A^2 + \|\delta p^{n-1/2}\|_A^2 - \|p^{n-1/2}\|_A^2) &\leq \\ -2\Delta t(\nabla \cdot \mathbf{u}^{n+1}, p^{*,n+1/2}) + \|\mathbf{u}^{n+1} - \mathbf{u}^n\|_{\mathbf{L}^2}^2 & \end{aligned}$$

which after summing with (14) gives

$$(18) \quad \begin{aligned} \|\mathbf{u}^{n+1}\|_{\mathbf{L}^2}^2 + \Delta t^2 \|p^{n+1/2}\|_A^2 + \frac{\Delta t}{2} (\|\nu^{1/2} \nabla \mathbf{u}^{n+1}\|_{\mathbf{L}^2}^2 + \frac{\Delta t}{2} \left\| \left( \frac{\nu}{k} \right)^{1/2} \mathbf{u}^{n+1} \right\|_{\mathbf{L}^2}^2) \\ \leq \|\mathbf{u}^n\|_{\mathbf{L}^2}^2 + \Delta t^2 \|p^{n-1/2}\|_A^2 + \frac{\Delta t}{2} \|\nu^{1/2} \nabla \mathbf{u}^n\|_{\mathbf{L}^2}^2 + \frac{\Delta t}{2} \left\| \left( \frac{\nu}{k} \right)^{1/2} \mathbf{u}^n \right\|_{\mathbf{L}^2}^2. \end{aligned}$$

The final estimate is obtained by summing (18) over the time levels from 0 to  $N-1 = T/\Delta t - 1$ .  $\square$

**3.2. Spatial discretization.** The equations (9–12) are discretized in space using the classical MAC stencil (see [8], figure 1). When discretizing the momentum equation in complex-shaped domains in the case of a MAC grid, it is necessary to take special care for computing the Brinkman term  $\nu \mathbf{u}/k$ , so that the accuracy is preserved. Essentially, such procedures compute some approximate or exact average of the coefficient  $\nu/k$  over the MAC cells intersected by a boundary marking a jump in this quantity (see for example [11], section 3.2).

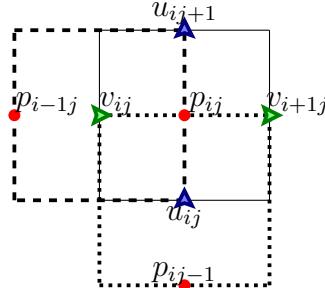


FIGURE 1. Control volumes associated with each node in case of MAC grid

The conservative finite volume approximation of the second derivative on a grid of a nonuniform grid size  $h_i$  is given by

$$(19) \quad D_x u_{i,j,k}^n = \frac{2}{h_{i+1} + h_i} \left( \bar{\nu}_{i-1/2} \frac{u_{i,j,k}^n - u_{i-1,j,k}^n}{h_i} - \bar{\nu}_{i+1/2} \frac{u_{i+1,j,k}^n - u_{i,j,k}^n}{h_{i+1}} \right),$$

where

$$(20) \quad \bar{\nu}_{i-1/2} = \left( \int_{-1}^0 \frac{ds}{\nu(x_i + sh_i, y, z)} \right)^{-1}$$

is the average of  $\nu$  on  $[x_{i,j,k}, x_{i-1,j,k}]$ , and  $h_i = x_{i,j,k} - x_{i-1,j,k}$ ,  $i = 2, \dots, L$ . Note that if the integral in (20) is approximated by means of the trapezoidal quadrature,  $\bar{\nu}$  would be equal to the harmonic average of  $\nu$  over a given discretization cell. The central difference in the  $y$  and  $z$  directions can be constructed in the same manner.

Similarly, the average Brinkman porosity at the point with x-coordinate  $x_i$  can be computed as:

$$(21) \quad \bar{k}_i = \left( \int_0^1 \frac{ds}{k(x_{i-1/2} + 0.5s(h_i + h_{i+1}), y, z)} \right)^{-1}$$

An alternative spatial approximation is provided by the stabilized central difference discretization on a collocated grid for the velocity and pressure proposed by Rhie and Chow [10]. All the test cases presented in the next section were also computed using such a collocated approximation and the results (not reported here) showed a very similar behaviour as the one computed on the MAC grid (presented below).

### 3.3. Computational cost of the direction splitting operator vs.

**Laplace operator in the pressure correction equation.** For simplicity, let us consider a cubic domain covered with a grid containing  $n \times n \times n$  nodes. Under the reasonable assumption that the cost for assembling the matrices is small compared to the cost of solving the systems of linear equations, let us evaluate the number of operations needed for computing the solution for the pressure *per time step*.

**Direction splitting operator.** In 3D, the Thomas algorithm for tri-diagonal matrices has to be employed  $3n^2$  times (i.e.,  $n^2$  times in each direction). Each Thomas algorithm requires  $5n$  multiplications/divisions and  $3n$  summations/subtractions. So, the total of  $15n^3 + 9n^3$  operations are required.

**Laplace operator.** Suppose now that we use an iterative method to solve the pressure Poisson equation <sup>2</sup>. Each iteration requires at least one matrix-vector multiplication, which for a seven-diagonal matrix will require  $7n^3$  multiplications and  $6n^3$  summations, thereby  $7n^3 + 6n^3$  is the total number of operation for one matrix-vector multiplication.

These results clearly indicate that even if the iterative method converges with only a couple of iterations, the solution of pressure equation with a direction splitting operator and a direct tri-diagonal solver requires less operations than the iterative solution of the pressure Poisson equation.

## 4. SIMULATIONS

**4.1. Preliminaries.** The performance of the two approximations of the incompressibility constraint discussed here: the pressure Poisson equation, and the directionally factorized perturbation (10), is compared on two two-dimensional problems involving pure fluid and porous regions. The domain of the first problem is a channel with sudden contraction, containing a subdomain (called here porous subdomain) with a given permeability. If the permeability is very large, the Darcy term in the Brinkman equations tends to zero, and therefore the flow in the porous subdomain is unrestricted similarly to the rest of the fluid domain. For very small values of the permeability, the Darcy term dominates in the porous subdomain, and completely prevents the flow through it. In fact, in this case the Stokes-Brinkman equations become a Fictitious Domain formulation for the Stokes equations with no-slip

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<sup>2</sup>Note that the Poisson equation can be solved using fast Fourier transform (FFT). The parallel performance of the FFT algorithm, however, is much worse than that of a Thomas-based Schur complement approach (see [6])

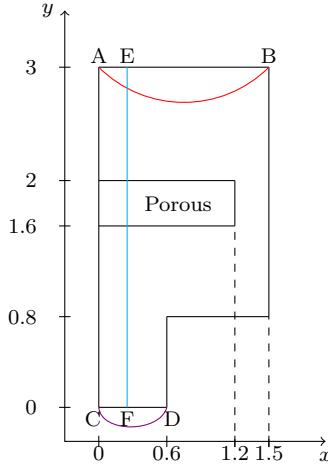


FIGURE 2. Sketch of a channel with a sudden contraction and a porous obstacle.

boundary conditions on the boundary of the "porous" subdomain. The domain of the second problem is a channel with two porous sub-domains. In this case time dependent boundary conditions are applied.

The main goal of these simulations is to compare the results for the velocity and pressure computed with the classical Poisson equation for the pressure increment (classical incremental projection scheme in a rotational form; see [5]), and the factorized operator  $A$  defined above. Therefore, we define  $\mathbf{u}_\Delta, p_\Delta$  to be the velocity and pressure calculated using the classical projection scheme, and  $\mathbf{u}_A, p_A$  – the velocity and pressure calculated using the scheme with the operator  $A$  in the pressure correction step.

In both test cases presented below the viscosity is set to  $\nu = 10^{-6} m^2 \cdot s^{-1}$ . The momentum equation for both schemes, and the pressure-Poisson equation in the classical projection scheme are solved by a generalized minimal residual method with ILU preconditioner (see [1] for implementation details). All simulations were performed on a machine with a dual core Intel Xeon 5148LV with 8 GB RAM .

**4.2. Flow in a channel with a sudden contraction and a porous obstacle.** As a first test case, consider the flow in a channel with a sudden contraction and a porous obstacle (see figure 2). No-slip boundary conditions are prescribed on the solid walls, while parabolic profile for the velocity at the inlet and the outlet is prescribed. The

usual zero Neumann boundary condition on the entire boundary is imposed on the pressure correction.

$$\begin{aligned} u_{inlet} &= \alpha_1 * x * (x - 1.5), \quad 0 \leq x \leq 1.5, \quad y = 3 \\ u_{outlet} &= \alpha_2 * x * (x - 0.6), \quad 0 \leq x \leq 1.5, \quad y = 0 \end{aligned}$$

Coefficients  $\alpha_1$  and  $\alpha_2$  are specified so that the flow rates at both ends of the channel are equal:

$$\int_{x1}^{x2} \alpha * (x - x_1) * (x - x_2) = Q, \text{ where } Q \text{ is the flow rate}$$

We show in figure 3 the  $L^2$  norm of the difference of the two velocities and the pressures,  $\|u_\Delta - u_A\|_{L_2}$  and  $\|p_\Delta - p_A\|_{L_2}$ , as a function of  $\Delta t$ . Clearly, the rate of decrease of both errors is similar to the theoretical estimates for the convergence error of the classical incremental projection scheme in a rotational form (see [7]): second order for the velocity and order 3/2 for the pressure error in the  $L^2$  norm. At the same time, the simulations with the directional splitting approach (using the factorized operator  $A$ ) are significantly faster (see table 1).

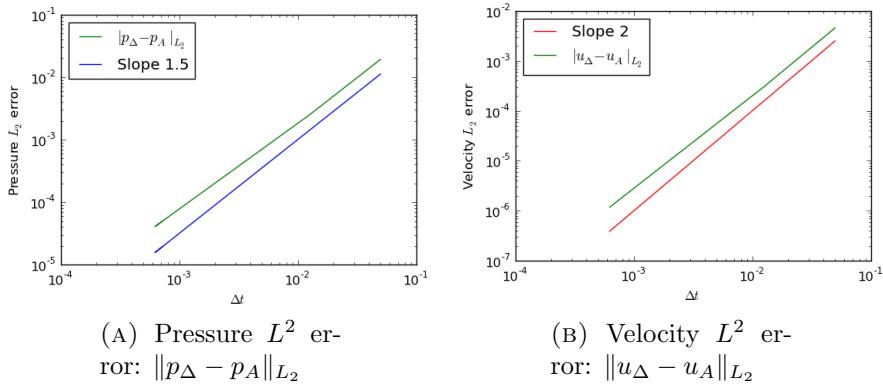


FIGURE 3. Pressure and velocity error; test case of section 4.2.

In the left panel of figure 4 we display the horizontal profiles of the velocity along the vertical segment EF (see figure 2) at various permeabilities. The corresponding velocity vectors are displayed in the right panel of figure 4.

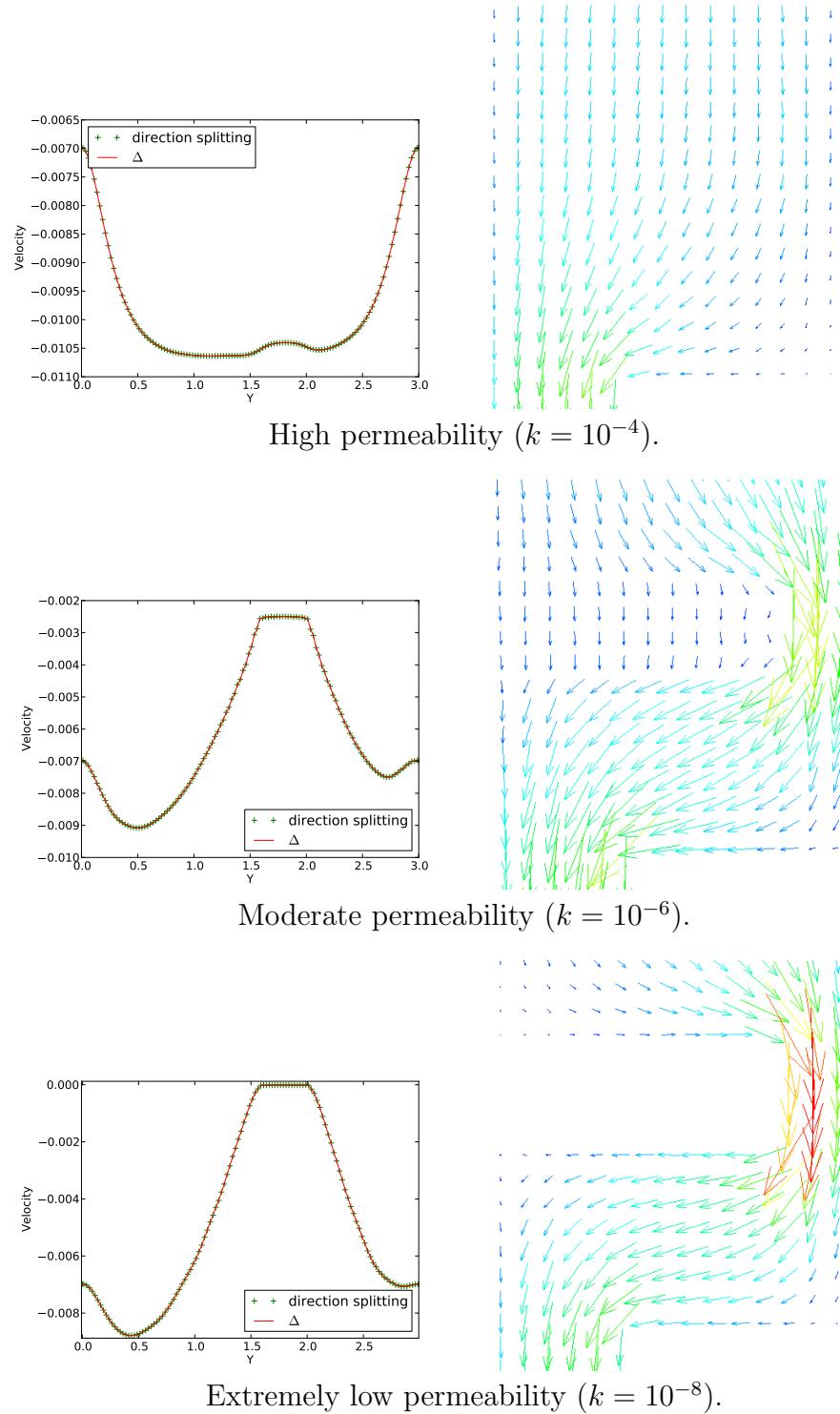


FIGURE 4. Velocity in the segment EF and the corresponding velocity field for different values of permeability; test case of section 4.2.

TABLE 1. CPU time comparison; test case of section 4.2

$h/N$	$\Delta t$	Tolerance for iterative solver	Time for Laplace	Time for DS operator	Speed up
0.01/37800	1e-5	1e-8	18.18	2.35	7.7
0.01/37800	1e-5	1e-12	34.12	2.35	14.52
0.01/37800	1e-7	1e-8	22.14	2.35	9.42
0.01/37800	1e-7	1e-12	28.89	2.35	12.3
0.005/151200	1e-5	1e-8	86.12	6.57	13.1
0.001/151200	1e-5	1e-12	97.43	6.57	14.8
0.001/151200	1e-7	1e-8	78.16	6.57	11.8
0.001/151200	1e-7	1e-12	84.25	6.57	12.8

**4.3. Flow in a channel with two porous obstacles and time dependent boundary conditions.** To further illustrate the properties of the factorized scheme, consider the flow in a vertical channel with two porous blocks (see figure 5). No-slip boundary conditions are prescribed on the solid walls, while time dependent profiles for the velocity at the inlet and the outlet are prescribed:

$$u_{inlet} = u_{outlet} = -(\pi + \sin(\alpha t)), \\ 0 \leq t \leq T, 0 \leq x \leq 1.5, 0 \leq y \leq 1$$

where  $\alpha$  is a frequency parameter.

In terms of figure 5, the segment AB is the inlet, and the segment CD is the outlet. In figure 6 we show the error on the velocity and the pressure as a function of  $\Delta t$ , and in figure 7 – the vertical profiles of the velocity along the vertical segment EF, as well as the velocity vectors in the entire domain. As in the previous test case, the velocity and pressure errors have a very similar convergence rate to the corresponding convergence errors of the classical incremental projection scheme in a rotational form.

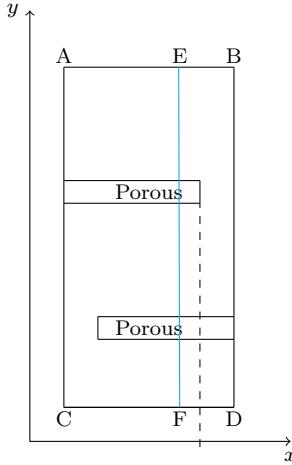


FIGURE 5. Sketch of the channel with two porous obstacles.

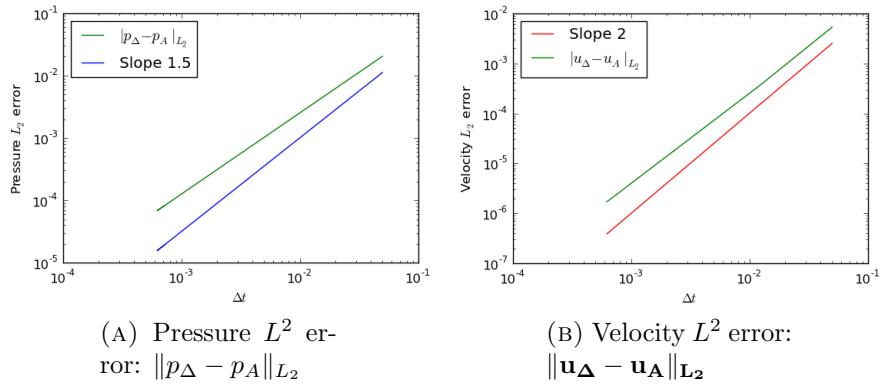
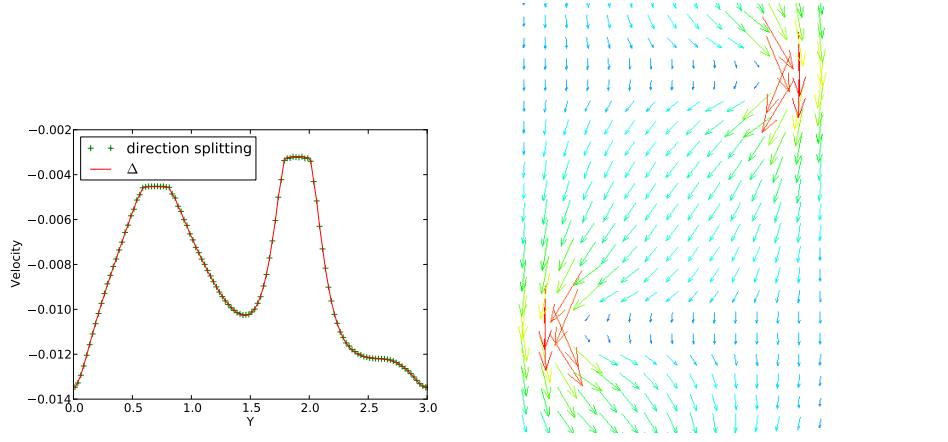
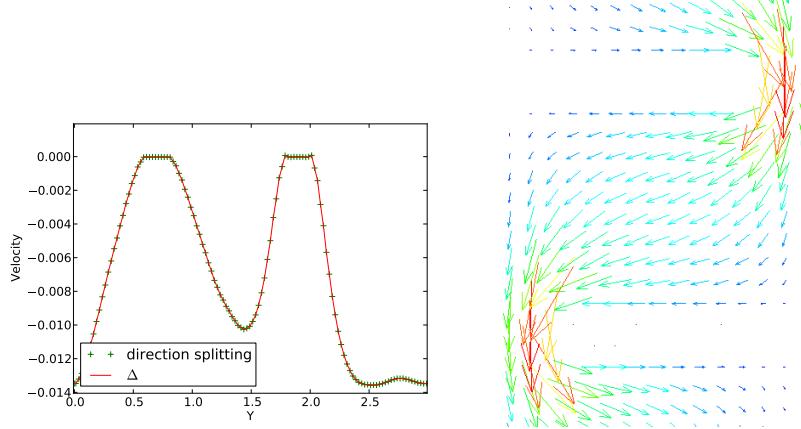


FIGURE 6. Pressure and velocity error; test case of section 4.3.

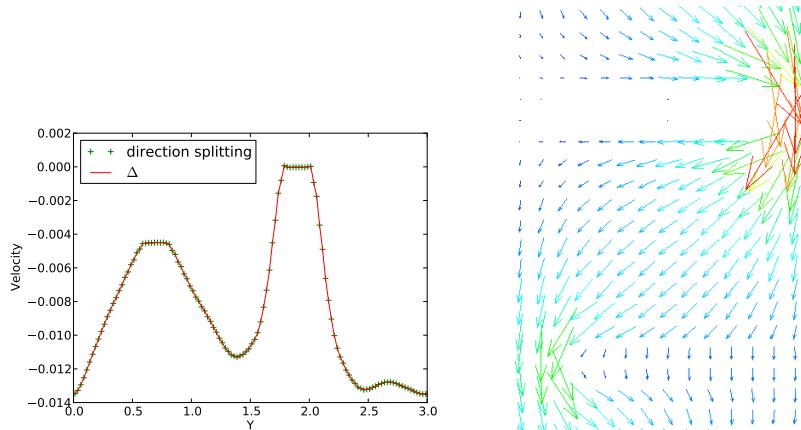
Table 2 shows the CPU time comparison for the two schemes, again the simulations with the directional splitting approach being significantly faster .



Moderate permeability in both porous areas ( $k_{upper} = k_{lower} = 10^{-6}$ ).



Extremely low permeability in both porous areas  
( $k_{upper} = k_{lower} = 10^{-8}$ ).



Low permeability in the upper area ( $k_{upper} = 10^{-8}$ ), and moderate permeability in the lower area ( $k_{lower} = 10^{-6}$ )

FIGURE 7. Velocity in the slice and corresponding velocity vectors for different values of permeability; test case of section 4.3.

TABLE 2. CPU time comparison; test case of section 4.3.

$h/N$	$\Delta t$	Tolerance for iterative solver	Time for Laplace	Time for DS operator	Speed up
0.01/45000	1e-5	1e-8	20.24	2.4	8.61
0.01/45000	1e-5	1e-12	38.12	2.4	16.2
0.01/45000	1e-7	1e-8	27.14	2.4	11.54
0.01/45000	1e-7	1e-12	33.89	2.4	14.42
0.005/180000	1e-5	1e-8	88.15	6.8	12.9
0.001/180000	1e-5	1e-12	99.4	6.8	14.6
0.001/180000	1e-7	1e-8	79.25	6.8	11.6
0.001/180000	1e-7	1e-12	85.13	6.8	12.5

## 5. CONCLUSIONS

The results presented in this article demonstrate that the direction-factorized perturbation of the incompressibility constraint in the Stokes equations proposed in [6] can be applied to the Stokes-Brinkman equations yielding an unconditionally stable scheme. We do not consider here the possibility for a direction splitting of the momentum equations since it would generally yield non-commutative one-dimensional operators which significantly complicates the stability estimate. Although some possibilities for such a splitting of the momentum equation in the non-commutative case are discussed in [3] the stability of the overall algorithm in the Stokes or Stokes-Brinkman case is still an open problem.

The presented numerical results clearly demonstrate on two test cases involving fluid and porous areas, that the difference between the velocities and pressures computed with the current direction-slitting approach, and the classical incremental projection scheme in a rotational form decrease as  $O(\Delta t^2)$  and  $O(\Delta t^{3/2})$  respectively. In other words, the convergence rates in time of the two algorithms are close to the theoretical estimates for the incremental projection scheme in a rotational form.

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